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The main objective of this research was the theoretical and experimental investigation of the physics of self-pumped and mutually-pumped phase conjugators and the development of novel applications. In addition, the scope of the research was expanded to include the effect of partial coherence (finite coherence length) on nonlinear wave mixing and phase conjugation in photorefractive media. Specifically, the effect of finite mutual coherence on the formation of index gratings, especially 2k-gratings, in two–wave mixing and self-pumped phase conjugation was studied.

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Studies of Nonlinear Optical Wave Mixing and Phase Conjugation with Partially Coherent Waves

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1.0 Research Description

The main objective of this research is to continue our theoretical and experimental investigation on physics of self-pumped and mutually-pumped phase conjugators and the development of novel applications. In addition, we also expand the scope of our research to include the effect of partial coherence (finite coherence length) on nonlinear wave mixing and phase conjugation in photorefractive media. This is an important and practical subject which has not been investigated by previous researchers. Specifically we are interested in the effect of finite mutual coherence on the formation of index gratings, especially 2k-gratings, in two-wave mixing and self-pumped phase conjugation. The same formulation and results can be applied to stimulated Brillouin scattering (SBS) in Kerr media. The original proposal requested a performance period of 36 months. Due to a budget cut at the AFOSR, our proposal was reduced to a 8-month program with a significantly lower funding level. During the research period only 8 months, we have carried out intensive theoretical investigation in several important areas in nonlinear optical wave mixing and phase conjugation, especially the problem of beam coherence. Specifically, we have investigated the following problems:

- Contradirectional two-wave mixing with partially coherent waves

- Two-wave mixing with partially coherent waves in high-speed photorefractive media
- Effect of partial coherence on phase conjugation

1.1 Scientific Problem

The unique distortion correction property of optical phase conjugation makes it extremely attractive and important in many advanced optical systems. These include free space optical communication, atmospheric correction and beam control of high power lasers, optical image processing, etc. There are many approaches which can be employed to achieve nonlinear optical phase conjugation. These include optical four-wave mixing (FWM), stimulated Brillouin scattering (SBS), self-pumped phase conjugation (SPPC), etc. In nonlinear media such as photorefractive crystals and Kerr media, it is possible to achieve self-pumped phase conjugation (SPPC) and mutually pumped phase conjugation (MPPC)[1,2]. Such phase conjugators are ideal for optical communications through scattering media such as atmosphere or sea water. MPPC can also be employed for the phase locking of an array of lasers.

Both SPPC and MPPC in photorefractive media have been investigated both theoretically and experimentally. Only transmission gratings were considered in the theoretical investigations for reasons of mathematical simplicity. However, in most experimental situations, both reflection gratings and 2k-gratings are present even when the coherent length of the laser is limited. A complete theory requires the inclusion of reflection gratings and 2k-gratings in the coupled equations. During the current research program, we have investigated the role of 2k-gratings and reflection gratings in photorefractive mutually pumped phase conjugators (MPPC). We discovered that, depending on the crystal orientation and the sign of the coupling constant, the presence of 2k-gratings and reflection gratings may enhance the phase conjugation efficiency [3,4]. In addition, the presence of these additional gratings may lead to temporal instability in phase conjugation due to grating competition.

Although reflection gratings and 2k-gratings can be neglected in some situations, they are significant in many important applications including phase locking of lasers using MPPC. In a preliminary model, we assume that the grating strength is directly

related to the normalized mutual coherence of the input beams. Thus, by properly arranging the path difference between the input beams, it is possible to control the mutual coherence, and thus the strength of the reflection gratings and 2k-gratings. For example, it is possible to use two mutually incoherent beams in MPPC in which both reflection gratings and 2k-gratings can be neglected. This is not entirely correct in SPPC. As a result of a preliminary study, we found that the mutual coherence of the two beams actually depends on the coupling strength. In other words, the mutual coherence can be enhanced or reduced by the coupling. This can lead to an effective interaction length (length of reflection gratings and/or 2k-gratings) which can be very different from the coherent length.

1.2 Scientific and Technical Approach

Nonlinear optical phase conjugation offers unique capabilities and possibilities for many new applications. These include optical communication through scattering media, wavefront matched heterodyne detection, wavefront correction and beam control of high power laser beams (e.g., ABL), optical image processing, etc. Although there have been numerous theoretical and experimental works in this area, technical issues such as reflection gratings, 2k-gratings, dynamic range and stability of the conjugators remain unresolved. In the following, we briefly describe the effect of 2k-grating in self-pumped phase conjugators. We then describe a theoretical model of the role of 2k-gratings in SPPC and a general formulation on the effect of mutual coherence on the strength of 2k-gratings in photorefractive media.

In self-pumped phase conjugators (SPPCs) the counterpropagating beams needed in a four-wave mixing process are generated by the incident beam itself. By a process that is not entirely understood, the self-generated pump beams are often formed in a configuration tending to optimize the phase conjugated beam. SPPCs have been demonstrated in a variety of configurations [2]. In SPPC involving no external mirrors, there are at least three different models available to explain the origin of the phase conjugation in a single photorefractive crystal. These are four-wave mixing with total internal reflection (FWM-TIR) [5,6], stimulated photorefractive backscattering (SPB) [7-9], four wave mixing and stimulated photorefractive backscattering (FWM-SPB) [10-14]. In the last model, the SPPC formation relies on both four wave mixing (FWM) and stimulated photorefractive backscattering (SPB). The phase conjugate beam is generated by a four-wave mixing process involving the incident beam, forward-propagating beam (the fanning beam) and its backward-stimulated scattering beam. In a manner very similar to stimulated Brillouin scattering (SBS), the backward-stimulated scattering beam is generated by the SPB process involving 2k-gratings. Hence, instead of a closed loop inside the crystal, filaments representing counter-propagating beams are often observed.

In a typical SPPC experimental configuration, an extraordinarily-polarized input beam is incident on the a-face of a regular-cut $BaTiO_3$ photorefractive crystal. The SPPC formation mechanism is usually determined by the fanning pattern and the boundary conditions[12]. Generally speaking, SPPC is often generated via the SPB or FWM-SPB mechanisms in some doped crystals, but via the FWM-TIR mechanism in undoped crystals. This is mainly due to a significantly stronger fanning and larger coupling coefficients for 2k-gratings in doped crystals. Changing the dopant concentration or the operating wavelength can cause a reconfiguration of the initial fanning pattern which leads to a change of the physical mechanism and the reflectivity of SPPC [12-15]. Depending on the boundary conditions, the fanned beam may or may not reach the corner of the crystal cube. SPPC via FWM-TIR often involves the participation of the crystal corner to provide the total reflection.

In the previous studies, we assumed that the two beams responsible for the 2k-gratings are mutually coherent. This is a good assumption provided that the coherence length of the laser is much larger than the crystal dimension. In the FWM-SPB mode of SPPC, the two beams are almost always coherent. This is not entirely true in some MPPCs and SPPCs. In the event when the coherent length of the laser is similar to (or even smaller than) the crystal dimension, the strength of the 2k-gratings will be reduced due to the reduction in the modulation depth (fringe contrast) of the interference pattern. Assuming no beam coupling, it is known that the modulation depth (interference fringe contrast) is proportional to the normalized mutual coherence of the two beams. In a photorefractive medium, the photo-induced index grating is proportional to the modulation depth (fringe contrast). Thus, it is nature to assume that the beam coupling constant is directly proportional to the mutual coherence of the two beams. In other words,

$$\gamma_2 = \gamma_{20}\Gamma_{12}(z) \tag{1}$$

where γ_{20} is the coupling constant for monochromatic beams, $\Gamma_{12}(z)$ is the mutual coherence of the two beams which is a function of position z. In a contradirectional two-wave mixing, $\Gamma_{12}(z)$ is significant only over a range of the coherence length. Thus, it is often assumed that the interaction length is limited to the coherence length. This approximation is only valid when the mutual coherence remains unchanged during the interaction. In the following section, we will show that the mutual coherence of the two interacting beams can be significantly changed as a result of the beam coupling, especially when the coupling constant is large.

Two-Beam Coupling without pump depletion

Two-wave mixing in photorefractive crystals has been investigated extensively by previous workers [1]. Most of the theoretical works in this area are on the interaction of two monochromatic beams. Two wave mixing with partially coherent beams has been studied recently for the case of transmission grating interaction only [19], for reasons of mathematical simplicity. In the case of transmission grating interaction, the optical path difference between the two interacting waves remains approximately the same as the two waves propagate codirectionally through the photorefractive medium, especially when the incident angles of the two waves are close to each other. In the case of reflection grating interaction, the optical path difference between the two interacting waves varies significantly as the two waves propagate contradirectionally through the photorefractive medium. This is the reason why the mutual coherence $\Gamma_{12}(z)$ depends on position. Thus, for the case of transmission grating interaction, only one free variable for the position is needed to describe in a self-consistent way the second order statistical properties of the two optical waves, i.e. their intensities and mutual coherence, while at least two free variables, one for the position and one for the optical path difference, will be needed for the case of reflection grating interaction. Another difficulty in studying the reflection grating interaction of partially coherent waves is to find a way to incorporate the complete boundary conditions into the theoretical formulation as a result of the two-point boundary value problem. In such a problem, a complete set of boundary conditions includes both the second order self statistical properties (e.g. self-coherence) of each wave at its entrance boundary and the second order mutual statistical properties (e.g. mutual coherence) of the two waves at their respective entrance boundaries. In this section, we present a preliminary theoretical investigation for the case of reflection grating interaction. In the case of contradirectional two wave mixing with purely

diffusion-driven photorefractive medium, the coupled-wave equations for the slowly varying amplitudes $\mathbf{E}_1(\mathbf{t},\mathbf{z})$ and $\mathbf{E}_2(\mathbf{t},\mathbf{z})$ can be written as [19]

$$\frac{\partial \mathbf{E}_1}{\partial \mathbf{z}}+\frac{1}{\mathbf{v}}\frac{\partial \mathbf{E}_1}{\partial \mathbf{t}}=\frac{\gamma}{2}\frac{\mathbf{Q}\cdot\mathbf{E}_2}{\mathbf{I}_1+\mathbf{I}_2}-\frac{\alpha}{2}\mathbf{E}_1 \tag{2}$$

$$\frac{\partial \mathbf{E}_2}{\partial \mathbf{z}'}-\frac{1}{\mathbf{v}}\frac{\partial \mathbf{E}_2}{\partial \mathbf{t}}=\frac{\gamma}{2}\frac{\mathbf{Q}^*\cdot\mathbf{E}_1}{\mathbf{I}_1+\mathbf{I}_2}+\frac{\alpha}{2}\mathbf{E}_2 \tag{3}$$

where γ is the intensity coupling constant, α is the intensity absorption coefficient, $\mathbf{v}$ is the group velocity, $\mathbf{E}_1\exp(-\mathbf{i}\omega\mathbf{t}+\mathbf{ikz})$ and $\mathbf{E}_2\exp(-\mathbf{i}\omega\mathbf{t}-\mathbf{ikz})$ are the coupled quasi-monochromatic waves which interact through a dynamic photorefractive grating $\delta\mathbf{n}(\mathbf{z},\mathbf{t})\propto\mathbf{Q}\exp[2\mathbf{ikz}]+\mathbf{c.c.}$. The dynamics of the photorefractive index grating can be written as

$$\tau\frac{\partial \mathbf{Q}}{\partial \mathbf{t}}+\mathbf{Q}=\mathbf{E}_1\mathbf{E}_2^* \tag{4}$$

where τ is the relaxation time constant determined by the average total intensity, $\tau\sim\left(\langle E_1\rangle^2+\langle E_2\rangle^2\right)^{-1}$. Reflecting the nature of partial coherence, the complex amplitudes of the two waves are considered random variables, especially in the regime when the coherence time $\delta\omega^{-1}$ is substantially less than the relaxation time of the material $\delta\omega\tau>>1$ [20]. When the optical path difference of the two beams is smaller than the coherence length of the laser, a dynamic photorefractive grating is recorded in the medium. Its position and profile are nearly stationary. Thus, as an approximation, we can replace the dynamic grating amplitude $\mathbf{Q}$ in Eqs.(2) and (3) with its ensemble average as $\mathbf{Q}\approx\langle\mathbf{E}_1\mathbf{E}_2^*\rangle$.

Since the complex amplitudes $\mathbf{E}_1(\mathbf{t},\mathbf{z})$ and $\mathbf{E}_2(\mathbf{t},\mathbf{z})$ are stationary random processes, we can define some of their ensemble averages as $\Gamma_{12}(\mathbf{z},\Delta\mathbf{t})\equiv\langle\mathbf{E}_1(\mathbf{z},\mathbf{t}_1)\mathbf{E}_2^*(\mathbf{z},\mathbf{t}_2)\rangle$, $\Gamma_{11}(\mathbf{z},\Delta\mathbf{t})\equiv\langle\mathbf{E}_1(\mathbf{z},\mathbf{t}_1)\mathbf{E}_1^*(\mathbf{z},\mathbf{t}_2)\rangle$, and $\Gamma_{22}(\mathbf{z},\Delta\mathbf{t})\equiv\langle\mathbf{E}_2(\mathbf{z},\mathbf{t}_1)\mathbf{E}_2^*(\mathbf{z},\mathbf{t}_2)\rangle$ where $\Delta\mathbf{t}=\mathbf{t}_1-\mathbf{t}_2$ and $\langle\ \rangle$ means ensemble average. We refer to Γ_{11} and Γ_{22} as the self coherence of the pump beam $\mathbf{E}_1(\mathbf{t},\mathbf{z})$ and the signal beam $\mathbf{E}_2(\mathbf{t},\mathbf{z})$ respectively, and Γ_{12} as the mutual coherence between the signal beam and the pump beam. With these notations, we can immediately write down $\mathbf{Q}\approx\Gamma_{12}(\mathbf{z},0)$, $\mathbf{I}_1(\mathbf{z})=\Gamma_{11}(\mathbf{z},0)$ and $\mathbf{I}_2(\mathbf{z})=\Gamma_{22}(\mathbf{z},0)$. Eqs.(2) and (3) can therefore be reduced to a system for these averaged values as

$$\frac{\partial\Gamma_{12}(\mathbf{z},\Delta\mathbf{t})}{\partial\mathbf{z}}=-\frac{2}{\mathbf{v}}\frac{\partial\Gamma_{12}(\mathbf{z},\Delta\mathbf{t})}{\partial\Delta\mathbf{t}}+\frac{\gamma}{2}\frac{\Gamma_{12}(\mathbf{z},0)}{\mathbf{I}_1+\mathbf{I}_2}[\Gamma_{11}(\mathbf{z},\Delta\mathbf{t})+\Gamma_{22}(\mathbf{z},\Delta\mathbf{t})] \tag{5}$$

$$\frac{\partial\Gamma_{11}(\mathbf{z},\Delta\mathbf{t})}{\partial\mathbf{z}} = \frac{\gamma}{2}\frac{\Gamma_{12}(\mathbf{z},0)}{\mathbf{I}_1+\mathbf{I}_2}\Gamma_{12}^*(\mathbf{z},-\Delta\mathbf{t}) + \frac{\gamma}{2}\frac{\Gamma_{12}^*(\mathbf{z},0)}{\mathbf{I}_1+\mathbf{I}_2}\Gamma_{12}(\mathbf{z},\Delta\mathbf{t}) - \alpha\Gamma_{11}(\mathbf{z},\Delta\mathbf{t}) \quad (6)$$

$$\frac{\partial\Gamma_{22}(\mathbf{z},\Delta\mathbf{t})}{\partial\mathbf{z}} = \frac{\gamma}{2}\frac{\Gamma_{12}(\mathbf{z},0)}{\mathbf{I}_1+\mathbf{I}_2}\Gamma_{12}^*(\mathbf{z},-\Delta\mathbf{t}) + \frac{\gamma}{2}\frac{\Gamma_{12}^*(\mathbf{z},0)}{\mathbf{I}_1+\mathbf{I}_2}\Gamma_{12}(\mathbf{z},\Delta\mathbf{t}) + \alpha\Gamma_{22}(\mathbf{z},\Delta\mathbf{t}) \quad (7)$$

Note that Eqs.(5)-(7) represent the propagation of the mutual conference and self coherence functions of the two beams. When there is no coupling (i.e. $\gamma = 0$), Eqs.(5)-(7) can be solved analytically. The solutions are consistent with the classical theory on optical wave interference in linear medium[21]. The mutual coherence function is presented as a propagation wave with the boundary conditions at z = 0 plane. The self coherence function decreases along the z axis due to absorption. When the coupling is present, both beams are scattered by the grating into each other. As a result, part of pump wave branches off in the direction of the signal waves. Thus, the mutual and self conference functions of the two waves can not be derived by the simple propagation of the boundary conditions. However, we note that Eqs.(5)-(7) are self-consistent. They can be solved numerically if we have the proper boundary conditions. In the presence of pump depletion, we can not obtain sufficient boundary conditions for typical situations. Therefore, although Eqs.(5)-(7) are general, they are not useful for the case of pump depletion. For the case of nondepleted pump, solutions with sufficient boundary conditions can be easily obtained. Preliminary results indicate that the contradirectional two wave mixing of partial coherent beams inside a photorefractive crystal with an inertial nonlinear response can lead to a increase of the mutual coherence substantially and the interaction length of the two beams. A set of the coupled equation has been derived to describe the propagation of the mutual coherence and the self coherence functions of the two beams.

Two-Beam Coupling with pump depletion

When beam depletion is not negligible, the investigation requires the use of statistical optical techniques, especially for the case of reflection gratings. In the above, we describe a theoretical formulation of the problem in the space and time domain for the reflection grating interaction in the nondepleted pump regime. By using the nondepleted pump approximation, we reduced the two-point boundary value problem to an initial value problem. In this section, we present a preliminary result of our general formulation of the problem in the space and frequency domain based on the standard statistical theory on linear systems. Specifically, we are interested in the signal intensity gain and the mutual coherence in the contradirectional wave mixing of two partially coherent waves. Photorefractive two-wave mixing is a nonlinear optical process. Because of the mutual coherence of the two waves, a dynamic holographic grating is formed in the medium. Its position and index profile are nearly stationary under the condition of a cw illumination. Both waves are scattered into each other by the presence of this index grating. Scattering of partially coherent waves by a stationary grating can be modeled with a statistical theory on linear systems [21]. An iterative procedure can subsequently be devised to obtain the final photorefractive grating profile from an initially arbitrary grating profile.

We consider the interaction of two counter-propagating waves with partial coherence in a photorefractive medium between z=0 and z=L. Reflecting the partial coherence, $\mathbf{E}_1(\mathbf{z},\mathbf{t})$ and $\mathbf{E}_2(\mathbf{z},\mathbf{t})$ are treated as stationary random variables. They

represent the random fluctuation of the amplitudes of the two waves. Let $\mathbf{E}_{11}(\mathbf{z},\Delta\omega)$ and $\mathbf{E}_{22}(\mathbf{z},\Delta\omega)$ denote the self spectral density functions of $\mathbf{E}_1(\mathbf{z},\mathbf{t})$ and $\mathbf{E}_2(\mathbf{z},\mathbf{t})$ respectively and $\mathbf{E}_{12}(\mathbf{z}_1,\mathbf{z}_2,\Delta\omega)$ be the cross spectral density function between $\mathbf{E}_1(\mathbf{z}_1,\mathbf{t})$ and $\mathbf{E}_2(\mathbf{z}_2,\mathbf{t})$. The spectral density functions and the corresponding coherence functions are Fourier transform pairs, i.e.

$$\Gamma_{11}(\mathbf{z},\tau) = \int \mathbf{E}_{11}(\mathbf{z},\Delta\omega)\exp(-\mathbf{i}\Delta\omega\tau)\mathbf{d}\Delta\omega \tag{8}$$

$$\Gamma_{22}(\mathbf{z},\tau) = \int \mathbf{E}_{22}(\mathbf{z},\Delta\omega)\exp(-\mathbf{i}\Delta\omega\tau)\mathbf{d}\Delta\omega \tag{9}$$

$$\Gamma_{12}(\mathbf{z},\tau) = \int \mathbf{E}_{12}(\mathbf{z},\mathbf{z},\Delta\omega)\exp(-\mathbf{i}\Delta\omega\tau)\mathbf{d}\Delta\omega \tag{10}$$

where $\Delta\omega = \omega - \omega_0$ with ω being a general frequency component of the waves. With the above notations and relations, the intensity of the two waves can be expressed as

$$\mathbf{I}_1(\mathbf{z}) \equiv \Gamma_{11}(\mathbf{z},0) = \int \mathbf{E}_{11}(\mathbf{z},\Delta\omega)\mathbf{d}\Delta\omega \tag{11}$$

$$\mathbf{I}_2(\mathbf{z}) \equiv \Gamma_{22}(\mathbf{z},0) = \int \mathbf{E}_{22}(\mathbf{z},\Delta\omega)\mathbf{d}\Delta\omega \tag{12}$$

and the mutual coherence of the two waves can be expressed as

$$\Gamma_{12}(\mathbf{z},0) = \int \mathbf{E}_{12}(\mathbf{z},\mathbf{z},\Delta\omega)\mathbf{d}\Delta\omega \tag{13}$$

Using these equations, the intensity of the waves as well as the mutual coherence can be obtained as soon as the spectral density functions are obtained. In what follows, we will derive the spectral density functions by using the statistical approach.

Using the above notations and definitions, we now begin our discussion on the photorefractive interaction. Inside the photorefractive medium, a dynamic index grating is generated. It can be written as

$$\delta\mathbf{n} = -\mathbf{i}\frac{\gamma}{2}\frac{\mathbf{c}}{\omega_0}\left[\frac{\mathbf{Q}(\mathbf{z},\mathbf{t})}{\mathbf{I}_0(\mathbf{z})}\exp(2\mathbf{i}\mathbf{k}_0\mathbf{z}) + \mathbf{c.c.}\right] \tag{14}$$

where $\mathbf{Q}(\mathbf{z},\mathbf{t})$ is a measure of the index grating, γ is the intensity coupling coefficient and $\mathbf{I}_0(\mathbf{z}) = \mathbf{I}_1(\mathbf{z}) + \mathbf{I}_2(\mathbf{z})$ is the total intensity at position z. For the purpose of our discussion, we will call $\mathbf{E}_1$ as the signal wave and $\mathbf{E}_2$ as the pump wave. Thus, for a photorefractive grating with a positive γ, the signal wave $\mathbf{E}_1$ can be amplified. If the

photorefractive effect is based purely on carrier diffusion (e.g. $BaTiO_3$), the dynamics of the index grating is described by Eq. (3). By virtue of photoexcitations, photorefractive processes are usually slow at low intensities. It is reasonable to assume that the coherence time $\delta\omega^{-1}$ of the two partially coherent waves is much smaller than the relaxation time τ of the photorefractive medium, i.e, $\delta\omega\tau >> 1$. Since $\mathbf{E}_1(\mathbf{z},\mathbf{t})$ and $\mathbf{E}_2(\mathbf{z},\mathbf{t})$ are stationary random variables, we can make the following approximation, according to Eq.(3),

$$\mathbf{Q}(\mathbf{z},\mathbf{t}) \cong \langle \mathbf{Q}(\mathbf{z},\mathbf{t}) \rangle = \Gamma_{12}(\mathbf{z},0) \tag{15}$$

In other words, two partially coherent waves with their complex amplitudes fluctuating randomly with time can actually write a stationary grating in a photorefractive medium under the appropriate conditions. For simplicity, we will denote $\mathbf{Q}(\mathbf{z},\mathbf{t})$ with $\mathbf{Q}(\mathbf{z})$ from now on. Note that $\mathbf{Q}(\mathbf{z})$ is also the mutual coherence of the two waves at position z. Given arbitrary functions of $\mathbf{Q}(\mathbf{z})$ and $\mathbf{I}_0(\mathbf{z})$, Eq.(14) yields an index grating. The propagation of a monochromatic wave through such an index grating can be described by the coupled wave equations. Let the general solutions be written $\mathbf{H}_{ij}(\mathbf{z},\omega)$ (i,j=1,2). We note that each input wave generates a transmitted and a reflected wave. The solutions can then be employed to modify the index grating as a result of the photorefractive effect. This leads to a new index profile $\mathbf{Q}(\mathbf{z})$. The iterations continue until the index profile converges.

In each of the iterations, an arbitrary stationary index grating can be considered as a linear system. We consider a given stationary index grating in the photorefractive medium as a linear system with the optical waves at the boundary planes z=0 and z=L as the input and the optical waves at an arbitrary plane z as the output. According to the theory on the statistical properties of linear systems, the second order statistical properties of the optical waves at the output plane can be expressed in terms of the second order statistical properties of the optical waves at the input planes and the frequency response of the linear system. Two physical processes happen simultaneously during two-wave mixing in a photorefractive medium. First, the two optical waves propagate through the photorefractive medium while being scattered into each other by the index grating. Second, the scattered waves modify the index grating through the photorefractive effect until a steady state is reached. We have provided above a mathematical model that describes these two physical processes separately. A steady state of the two-wave mixing in the photorefractive medium is reached when the two optical waves scattered by the photorefractive grating can exactly sustain the same photorefractive grating. A steady state solution of the two-wave mixing in a photorefractive medium can thus be obtained by using the mathematical model described above through an iterative procedure. The research results in this program are based on the statistical approach described above.

1.3 Publications:

1.3.1 Papers Published:

X. Yi, C. Yang, P. Yeh, S. Lin, K. Y. Hsu, "General solution of contradirectional two-wave mixing with partially coherent waves in

photorefractive crystals, " J. Opt. Soc. Am., B, Vol. 14, No. 6, 1396-1406 (1997).

X. Yi and P. Yeh, "Two-wave mixing with partially coherent waves in high-speed photorefractive media, " J. Opt. Soc. Am., B, Vol. 14, No. 11, 2885-2894 (1997).

X. Yi and P. Yeh, "Effect of partial coherence on phase conjuagtion," Opt. Comm., Vol. 147, N1-3, 126-130 (1998).

1.3.2 Conference Papers:

X. Yi and P. Yeh, "Phase conjugation through turbulent atmosphere," 1997 OSA Annual Meeting (Long Beach, CA, October 13-17, 1997), paper MJJ4.

R. Wang and P. Yeh, "Optical power limiting via both two-wave mixing and beam fanning in BaTiO3 crystals," 1997 OSA Annual Meeting (Long Beach, CA, October 13-17, 1997), paper ThFF24.

X. Yi, P. Yeh and S. Campbell, "Effect of thermal expansion on holographic memory," 1997 OSA Annual Meeting (Long Beach, CA, October 13-17, 1997), paper FK4.

P. Yeh, "Nonlinear Optical Wave Mixing with Partially Coherent Waves," (Invited Paper) Waseda International Symposium on Phase Conjugation and Wave Mixing, June 9-10, 1997 (Waseda University, Tokyo, Japan).

P. Yeh, "Recent Advances in Photorefractive Nonlinear Optics," (Invited Paper) CLEO/Pacific Rim'97 (July 14-18, Tokyo, Japan, 1997).

2.0 Progress:

2.1 Progress Summary:

2.1.1 Contradirectional two-wave mixing with partially coherent waves

During the period we investigated the contradirectional two-wave mixing with partially coherent waves in photorefractive media. We first considered the case when the material response is slow. This is typical of the photorefractive crystals with typical laser powers. By use of the statistical method described above, a general solution of the problem in the space and frequency domain is obtained. We obtained results on beam intensity and mutual coherence. The results on the enhancement of beam coherence are compared with previous theoretical results on simpler cases and with experimental measurements. Excellent agreements are achieved. The results also indicate that the effective interaction length can be significantly longer than the coherence length of the waves.

2.1.2 Two-wave mixing in high speed media

In the case when the laser power is very high, the material response is proportionally fast. We investigated the coupling of two partially coherent waves in photorefractive

media with a fast temporal response. When the coherence time of the waves is close to the speed of the photorefractive media, the photo-induced index grating will follow the amplitude fluctuation of the optical waves. The stochastic coupled wave equations are solved in the non-depleted pump region. The effects on the evolution of the signal wave are obtained.

2.1.3 Effect of partial coherence on phase conjugation

During the period, we also investigated the effect of partial temporal coherence of the incident wave on the phase conjugation process based on 2k-grating. In many photorefractive phase conjugators, 2k-gratings are generated by the incident beam and the phase conjugated beam. The phase conjugated beam will be amplified by the incident beam via contra-directional two-wave mixing. The strength of the 2k-grating and therefore the phase conjugate reflectivity will be dependent on the coupling between them. We obtained both theoretical and experimental results of the effect of beam coherence on the phase conjugate reflectivity.

2.2 Progress details:

Since most of the details of research results have already been published in technical journals. Detail discussion on the research results can also be found in Section 4 which contains reprints of the papers published.

3.0 References

1. See, for example, P. Yeh, *Introduction to Photorefractive Nonlinear Optics*, (Wiley, 1993).
2. See, for example, P. Yeh, "Photorefractive Phase Conjugators," Proc. IEEE, Vol. **80**, 435 (1992).
3. Q. B. He and P. Yeh, "Photorefractive mutually pumped phase conjugation with partially coherent beams," Appl. Phys. B 60, 47-50 (1995).
4. S. De La Cruz, S. MacCormick, J. Feinberg, Q. B. He, H. K. Liu and P. Yeh, "Effect of beam coherence on mutually pumped phase conjugators," J. Opt. Soc. Am. B, 12, 1363-1369 (1995).
5. J. Feinberg, "Self-pumped, continuous-wave phase conjugator using internal reflections," Opt. Lett. **42**, 919(1983).
6. K. R. MacDonald and J. Feinberg, "Theory of a self-pumped phase conjugator with two coupled interaction regions," J. Opt. Soc. Am. **73**, 548 (1983).
7. T. Y. Chang and R. W. Hellwarth, "Optical phase conjugation by backscattering in barium titanate," Opt. Lett. **10**, 408 (1985).
8. J. F. Lam, "Origin of phase conjugate waves in self-pumped photorefractive mirrors," Applied Physics Letters," Appl. Phys. Lett. **46**, 909 (1985)
9. G. C. Valley, "Evolution of phase-conjugate waves in stimulated photorefractive backscattering," J. Opt. Soc. Am. B **9**, 1440 (1992).
10. Y. W. Lian, H. Gao, P. Ye, Q. Guan and J. Wang, "Self-pumped phase conjugation with a new mechanism in $KTa_{1-x}Nb_xO_3$:Fe crystals", Appl. Phys. Lett. **63**, 1754 (1993).
11. Y. W. Lian, S. X. Dou, H. Gao, Y. Zhu, X. Wu, C. Yang and P. Ye, "Mechanism transformation with wavelength of self-pumped phase conjugation in $BaTiO_3$:Ce", Opt. Lett. **19**, 610 (1994)
12. A. A. Zozulya, M. Saffman, D. Z. Anderson, "Propagation of light beams in photorefractive media: fanning, self-bending, and formation of self-pumped four-wave-mixing phase conjugation geometry. Physical Review Letters, 8 Aug., Vol. 73, No. 6, 818-821(1994)
13. Y. W. Lian, S. X. Dou, Jiasen Zhang, H. Gao, Y. Zhu, X. Wu, C. Yang and P. Ye, "Variation of mechanism transition wavelength of self-pumped phase conjugation with Ce-content in $BaTiO_3$:Ce crystals," Optics Comm. **110**, 192 (1994)
14. Y. W. Lian, H. Gao, S. X. Dou, H. Wang, P. Ye, Q. Guan and J. Wang, "Mechanism transition of self-pumped phase conjugation in $KTa_{1-x}Nb_xO_3$:Fe crystals," ,Appl. Phys. B **59**, 655-685 (1994)
15. Y. W. Lian, S. H. Lin, S. Campbell, Ken Y. Hsu, Pochi Yeh and Yong Zhu, "Polarization-dependent mechanism transformation during self-pumped phase conjugation in $BaTiO_3$:Ce," Opt. Lett. **20**, 1683(1995).
16. P. Yeh, "Two-wave mixing in nonlinear media," IEEE J. Quantum Electron. **QE-25**, 484-519(1989).
17. Q. He, "Theory and applications of four-wave mixing in photorefractive media," IEEE J. Quantum Electron. **QE-24**, 2507(1988).
18. G. L. Wood, E. J. Sharp, and G. J. Salamo, "Performance of photorefractive self-pumped phase conjugators," Proc. SPIE **1626**, 21(1992).
19. N. V. Bogodaev, L.I. Ivleva, A. S. Korshunov, N. M. Polozkov, and V. V. Shkunov, J. Opt. Soc. Am. B **10**, 2287 (1993).
20. P. Yeh, Appl. Opt. **26**, 602 (1987).
21. See, for example, J. W. Goodman, *Statistical Optics*, ch. **5** (Wiley, New York, 1985).

4.0 Reprints of papers published

Two-wave mixing with partially coherent waves in high-speed photorefractive media

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We investigate the coupling of two partially coherent waves in high-speed photorefractive media. When the coherence time of the optical waves is close to the speed of the photorefractive media, the photoinduced index grating will follow the amplitude fluctuation of the optical waves. The stochastic coupled-wave equations are solved in the nondepleted pump region. The effects on the evolution of the signal wave are presented and discussed. © 1997 Optical Society of America [S0740-3224(97)04811-X]

1. INTRODUCTION

Nonlinear propagation and interaction of partially coherent optical waves in photorefractive materials have received a lot of attention in recent years, because of applications including image amplification, optical interconnection, optical phase conjugation, and photorefractive filters.[1–6] In most previous theoretical analysis, photorefractive materials have been assumed to have a long response time τ so that their photorefractive nonlinearity does not follow the instantaneous fluctuation of the partially coherent optical waves.[7–15] However, the response time of the photorefractive materials depends both on the materials themselves and on the intensities of the interacting waves.[16] Furthermore, whether the instantaneous photorefractive nonlinear response can be neglected depends on the relative photorefractive response time with respect to the bandwidth $\delta\omega$ of the interacting waves rather than on the photorefractive response time itself. The assumption of neglecting the instantaneous photorefractive response is valid mainly for low optical intensities and wide optical bandwidths so that the condition $\delta\omega\tau \gg 1$ is satisfied. When the optical intensity is high and the optical bandwidth is narrow so that $\delta\omega\tau$ is of the order of unity, the photorefractive nonlinearity will respond to the instantaneous fluctuation of the partially coherent optical waves. The nonlinear optical properties in the high-speed case are significantly different from those in the low-speed case.

In this paper we investigate codirectional two-wave mixing with partially coherent waves in high-speed photorefractive media. When the spectral bandwidth of the optical waves is close to the speed of the photorefractive media, the photoinduced index grating will fluctuate with the partially coherent optical waves. This case is much more complicated than the corresponding case in low-speed photorefractive media. To simplify the problem, we limit our analysis to the nondepleted pump region and assume that the pump wave has only phase noise. The effects on the evolution of the signal wave are determined. The results show that beam coupling can still be achieved for incoherent beams provided that the material reacts fast enough. It is important to distinguish wave mixing due to shared noise gratings[9–11] from the direct coupling of incoherent beams described here.

This paper is divided into three sections. In Section 2, a set of stochastic coupled-wave equations is introduced along with the necessary assumptions that ensure their validity in the high-speed case.[17] In Section 3, the stochastic coupled-wave equations are reduced to nonstochastic differential equations, from which the signal intensity and the mutual coherence of the two waves are obtained. In Section 4, the stochastic coupled-wave equations are rewritten into a series form, from which an asymptotic solution of the signal wave bandwidth is obtained.

2. THEORETICAL MODEL

In this section we present a theoretical model of wave mixing in the medium. Several major assumptions are made and discussed as we derive the theoretical model.

The system configuration is shown in Fig. 1. The signal wave and the pump wave enter the photorefractive medium at the incidence plane $z = 0$. The bisector of the two beams is perpendicular to the surface normal. A transmission grating with grating wave vector along the x direction is generated inside the photorefractive medium. The two optical waves are coupled through the index grating.

We assume that the two optical waves are plane waves of infinite extent in the transverse dimension. This is the first major assumption we make. Under this assumption the evolution of the optical-wave amplitudes can be confined to only one dimension. Thus, the optical waves inside the photorefractive medium can be written as

$$\begin{aligned}\mathbf{E}(z, t) = {} & E_1(z, t)\exp(-i\omega_0 t + i\mathbf{k}_{0z}z - i\mathbf{k}_{0x}x) \\ & + E_2(z, t)\exp(-i\omega_0 t + i\mathbf{k}_{0z}z + i\mathbf{k}_{0x}x),\end{aligned} \tag{1}$$

where $\mathbf{E}(z, t)$ is the total electric field of the two waves, E_1 is the signal wave complex amplitude, E_2 is the pump-wave complex amplitude, ω_0 is the center frequency of the

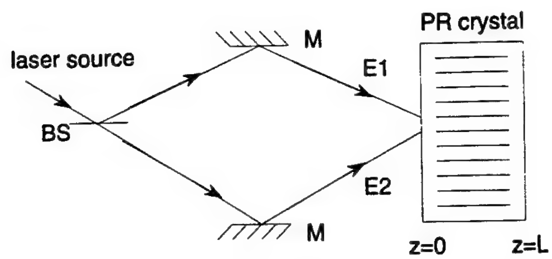

Fig. 1. System configuration: codirectional two-wave mixing with partially coherent waves.

two partially coherent waves, $\mathbf{k}_0 = n\omega_0/c$ is the center-wave vector, n is the refractive index of the photorefractive medium, and c is the speed of light in vacuum. Reflecting the partial coherence, $E_1(z, t)$ and $E_2(z, t)$ are stationary random variables. They represent the random fluctuation of the amplitudes of the two waves.

The instantaneous intensities of the two waves are $E_1(z, t)E_1{}^*(z, t)$ and $E_2(z, t)E_2{}^*(z, t)$. In general, they fluctuate randomly with time, e.g., as in thermal light and multimode laser light. The intensities of the two waves are defined as the ensemble average of their respective instantaneous intensities:

$$I_1(z) \equiv \langle E_1(z, t)E_1{}^*(z, t)\rangle, \tag{2}$$

$$I_2(z) \equiv \langle E_2(z, t)E_2{}^*(z, t)\rangle. \tag{3}$$

The intensities of the two waves as defined by Eqs. (2) and (3) are always constant over time. There is a special case. When a partially coherent optical wave has only phase noise, its instantaneous intensity is a constant over time and is equal to its ensemble averaged intensity, e.g., for single-mode laser light. The instantaneous mutual coherence of the two waves is $E_1(z, t)E_2{}^*(z, t)$, which also fluctuates randomly with time. The mutual coherence of the two waves is defined as the ensemble average of the instantaneous mutual coherence of the two waves.

We now consider the photorefractive grating dynamics. In the case of two-wave mixing with partially coherent waves in low-speed photorefractive media, the photorefractive grating can be written as

$$\delta n(z, t) = -i\frac{\gamma}{2}\frac{c}{\omega_0}\frac{\langle E_1(z, t)E_2{}^*(z, t)\rangle}{I_1(z) + I_2(z)} \times \exp(-2i\mathbf{k}_{0x}x) + \mathrm{cc}, \tag{4}$$

where γ is the intensity coupling coefficient. The amplitude of the photorefractive grating is proportional to the ensemble averaged mutual coherence of the two waves and is inversely proportional to the ensemble averaged total intensity of the two waves. In low-speed photorefractive media the photorefractive nonlinearity does not respond to the instantaneous fluctuations of the partially coherent optical waves. Equation (4) is a valid approximation for most kinds of partially coherent wave, e.g., thermal light, multimode laser light, and single-mode laser light. In high-speed photorefractive media, the photorefractive nonlinearity responds to the instantaneous fluctuation of the partially coherent optical waves. Both the fluctuation of the instantaneous mutual coherence of the two waves and the fluctuation of the instantaneous total intensity of the two waves need to be taken into account to allow us to determine the photorefractive grating. This is a difficult task in general. To overcome this difficulty, we introduce two more major assumptions. One is that the pump wave is much stronger than the signal wave and is undepleted in the course of the interaction. The other is that the pump wave has only phase noise at the incidence plane. Under these two assumptions, the pump wave will have only phase noise throughout the photorefractive medium. The instantaneous total intensity of the two waves is approximately equal to the instantaneous intensity of the pump wave and is a constant over time throughout the photorefractive medium. We now need only to consider the instantaneous mutual coherence of the two waves to determine the photorefractive grating. The photorefractive grating can be written as

$$\delta n(z, t) = -i\frac{\gamma}{2}\frac{c}{\omega_0}\frac{Q(z, t)}{I_2(z)}\exp(-2i\mathbf{k}_{0x}x) + \mathrm{cc}, \tag{5}$$

where $Q(z, t)$ is the photorefractive grating amplitude factor. In a photorefractive medium of purely diffusive type, the dynamics of the photorefractive grating can be written as

$$\tau\frac{\partial Q(z, t)}{\partial t} + Q(z, t) = E_1(z, t)E_2{}^*(z, t), \tag{6}$$

where τ is the photorefractive grating relaxation time constant. Equation (6) can be written in an integral form,

$$Q(z, t) = \frac{1}{\tau}\int_{-\infty}^{t} E_1(z, t')E_2{}^*(z, t')\exp\left(\frac{t' - t}{\tau}\right)\mathrm{d}t'. \tag{7}$$

Equation (7) is mathematically equivalent to Eq. (6) and is sometimes more convenient to use than Eq. (6) in our analysis.

With the photorefractive grating given by Eq. (5) and the undepleted pump assumption, we can write the coupled wave equations as

$$\frac{\partial E_1(z, t)}{\partial z} + \frac{1}{v}\frac{\partial E_1(z, t)}{\partial t} = \frac{\gamma}{2}\frac{Q(z, t)}{I_2(z)}E_2(z, t), \tag{8}$$

$$\frac{\partial E_2(z, t)}{\partial z} + \frac{1}{v}\frac{\partial E_2(z, t)}{\partial t} = 0, \tag{9}$$

where v is the group velocity of the two waves in the photorefractive medium. In arriving at Eqs. (8) and (9), we have also assumed that there is no absorption in the photorefractive medium. This is the fourth major assumption needed to simplify the analysis. A linear absorption can later be incorporated into the coupled wave equations.

In the above derivation we have made no assumption about the statistical properties of the signal wave. The signal wave is allowed to have both intensity noise and phase noise at the incidence plane. In addition, the statistical properties of the signal wave are also allowed to change in the course of interaction. The assumption that the pump wave at the incidence plane has only phase noise can be justified with typical experimental condi-

tions. According to Ref. 16, the photorefractive grating relaxation time constant decreases as the total intensity of the two interacting waves increases. When the total intensity is low, e.g., 1 W/cm^2, the grating relaxation time constant falls into the range from 1 ms to 1 s. When the total intensity is high, the grating relaxation time constant can be of the order of 1 μs or lower. The bandwidth of an optical wave whose coherence time is close to the grating relaxation time at the short end will be of the order of 1 MHz. This is a narrow bandwidth even for a single-mode laser source. The assumption that the pump wave at the incidence plane has only phase noise agrees well with experiments in which the pump wave is obtained from a single-mode laser. As we shall see, this assumption and the undepleted pump assumption lead to significant simplifications for solving the coupled-wave equations.

To solve the coupled-wave equations, we need to know certain information about the statistical properties of the signal wave and the pump wave at the incidence plane as the boundary condition. Without loss of generality, we assume that the signal wave and the pump wave are obtained from the same source laser wave. We also assume a single-mode laser source and that the source laser wave has only phase noise. The optical wave amplitudes $E_1(z, t)$ and $E_2(z, t)$ at the incidence plane can thus be written

$$E_1(0, t) = \sqrt{I_1(0)}\exp[-i\theta(t + t_d)], \tag{10}$$

$$E_2(0, t) = \sqrt{I_2(0)}\exp[-i\theta(t)], \tag{11}$$

where t_d is the time delay between the signal wave and the pump wave at the incidence plane and $\theta(t)$ is a random variable representing the random phase fluctuation of the optical-wave amplitudes at the incidence plane. The random phase $\theta(t)$ is assumed to have stationary increments [i.e., $\theta(t + \delta t) - \theta(t)$ is statistically independent of the time t]. Furthermore, the increment over an arbitrarily given time period δt, i.e., $\Delta\theta(\delta t) = \theta(t + \delta t) - \theta(t)$, is assumed to have a Gaussian distribution. The random phase shifts over nonoverlapping time intervals are assumed to be statistically independent. Under these assumptions, the source laser wave has a Lorentzian line shape, i.e., the typical line shape of a single-mode semiconductor laser. The variance of the random phase shift over an arbitrarily given time period δt is related to the bandwidth of the source laser wave $\delta\omega$ as

$$\langle \Delta\theta(\delta t)\rangle^2 = \delta\omega\,\delta t. \tag{12}$$

The above assumptions for the source laser wave are typical for a single-mode laser source.

The theoretical model is complete with the coupled wave equations (7)–(9) and the assumptions for the statistical properties of the signal wave and the pump wave at the incidence plane as the boundary condition.

3. SIGNAL INTENSITY AND MUTUAL COHERENCE

In this section we solve for the signal intensity and the mutual coherence of the two waves. The nonlinear stochastic partial differential equations (7)–(9) are reduced to nonstochastic differential equations, the solutions of which include the signal intensity and the mutual coherence of the two waves. This section is further divided into three subsections.

A. Solutions

In this subsection the coupled-wave equations (7)–(9) are solved, and the expressions for the signal intensity and the mutual coherence of the two waves are obtained along with the necessary boundary conditions.

We first look at the pump wave. At the incidence plane, the pump wave has only phase noise. With Eq. (9), the amplitude of the pump wave inside the photorefractive medium can be written as

$$E_2(z, t) = E_2(0, t - z/v). \tag{13}$$

The pump wave propagates through the photorefractive medium without changing its statistical properties. Therefore the pump wave has the same intensity and has only phase noise throughout the photorefractive medium, i.e.,

$$I_2(z) = I_2(0), \tag{14}$$

$$E_2(z, t)E_2^*(z, t) = I_2(0). \tag{15}$$

Note that the instantaneous intensity $E_2(z, t)E_2^*(z, t)$ is a constant equal to the ensemble-averaged intensity when $E_2(z, t)$ has only phase fluctuations.

Second, we consider the mutual coherence of the two waves. We denote the instantaneous mutual coherence of the two waves as

$$F(z, t) \equiv E_1(z, t)E_2^*(z, t). \tag{16}$$

Using Eqs. (7)–(9), and (15), we obtain

$$\frac{\partial F(z, t)}{\partial z} + \frac{1}{v}\frac{\partial F(z, t)}{\partial t} = \frac{\gamma}{2}\frac{1}{\tau}\exp\left(-\frac{t}{\tau}\right)\int_{-\infty}^{t} F(z, t') \times \exp\left(\frac{t'}{\tau}\right)dt'. \tag{17}$$

This equation governs the propagation of the instantaneous mutual coherence of the two waves in the photorefractive medium. Note that Eq. (17) is a linear equation of the random variable $F(z, t)$. Taking the ensemble average of Eq. (17), we obtain

$$\langle F(z, t)\rangle = \langle F(0, t)\rangle \exp\left(\frac{\gamma z}{2}\right). \tag{18}$$

$\langle F(z, t)\rangle$ is the ensemble-averaged mutual coherence of the two waves. According to Eq. (18), the mutual coherence of the two waves increases exponentially with the position z.

We now define a fourth-order coherence function of the optical-wave amplitudes $E_1(z, t)$ and $E_2(z, t)$ as

$$R(z, \Delta t) \equiv \langle E_1(z, t + \Delta t)E_2^*(z, t + \Delta t)E_1^*(z, t) \times E_2(z, t)\rangle. \tag{19}$$

According to Eqs. (15) and (19), we have

$$I_1(z) = \frac{R(z, 0)}{I_2(0)}. \tag{20}$$

Note that the function $R(z, \Delta t)$ is the autocorrelation function of the instantaneous mutual coherence of the two waves, i.e.,

$$R(z, \Delta t) = \langle F(z, t + \Delta t)F^*(z, t)\rangle. \tag{21}$$

As the time delay Δt approaches infinity, the two random variables $F(z, t + \Delta t)$ and $F(z, t)$ become uncorrelated. According to Eq. (21), we have

$$R(z, \infty) = |\langle F(z, t)\rangle|^2. \tag{22}$$

In other words, $R(z, \infty)$ is the squared mutual coherence of the two waves. Using Eq. (18), we can express $R(z, \infty)$ explicitly.

With Eq. (20), the problem of solving for the signal intensity is reduced to the problem of solving for the autocorrelation function of the instantaneous mutual coherence of the two waves. Using Eqs. (17) and (21), we obtain

$$\frac{\partial R(z, \Delta t)}{\partial z} = \frac{\gamma}{2}\frac{1}{\tau}\int_{-\infty}^{+\infty} R(z, \Delta t + \Delta t')\exp\left(-\frac{|\Delta t'|}{\tau}\right)\mathrm{d}\Delta t'. \tag{23}$$

We define the Fourier transform of the function $R(z, \Delta t)$ as

$$\tilde{R}(z, \xi) = \frac{1}{2\pi}\int R(z, \Delta t)\exp(i\xi\Delta t)\mathrm{d}\Delta t. \tag{24}$$

Using Eqs. (23) and (24), we obtain

$$\tilde{R}(z, \xi) = \tilde{R}(0, \xi)\exp\left(\frac{\xi z}{1 + \xi^2\tau^2}\right). \tag{25}$$

Given the boundary condition $R(0, \Delta t)$, we can obtain $R(z, \Delta t)$ with Eqs. (24) and (25).

Using Eqs. (10) and (11), the boundary condition $R(0, \Delta t)$ can be written as

$$R(0, \Delta t) = I_1(0)I_2(0)\langle\exp[i\theta(t + \Delta t + t_d) - i\theta(t + \Delta t) - i\theta(t + t_d) + i\theta(t)]\rangle. \tag{26}$$

The above expression involves the random phase $\theta(t)$ at four different times: $t + \Delta t + t_d$, $t + \Delta t$, $t + t_d$, and t. The temporal sequence of these times plays an important role in determining the explicit expression of the function $R(0, \Delta t)$. As is shown in Fig. 2, there are four possible sequences for $t_d > 0$ and four possible sequences for $t_d < 0$. As an example, we consider one such sequence, the case $\Delta t > t_d > 0$. The random phase shift $\Delta\theta'(t_d) = \theta(t + \Delta t + t_d) - \theta(t + \Delta t)$ and the random phase shift $\Delta\theta''(t_d) = \theta(t + t_d) - \theta(t)$ are over nonoverlapping time intervals, and therefore they are statistically independent. Thus we can simplify Eq. (26) as, for this particular case,

$$R(0, \Delta t) = I_1(0)I_2(0)\langle\exp[i\Delta\theta'(t_d)]\rangle\langle\exp[-i\Delta\theta''(t_d)]\rangle. \tag{27}$$

Using Eq. (12) and the assumption that the random phase shift $\Delta\theta(\delta t)$ over an arbitrarily given time period δt has a Gaussian distribution, we obtain

$$\begin{aligned} R(0, t) &= I_1(0)I_2(0)\exp[-\langle(\Delta\theta'(t_d)^2\rangle/2] \\ &\quad \times \exp[-\langle(\Delta\theta''(t_d)^2\rangle/2] \\ &= I_1(0)I_2(0)\exp(-\delta\omega t_d). \end{aligned} \tag{28}$$

The boundary condition $R(0, \Delta t)$ for the other cases can be obtained similarly. The general result for the complete boundary condition can be written as

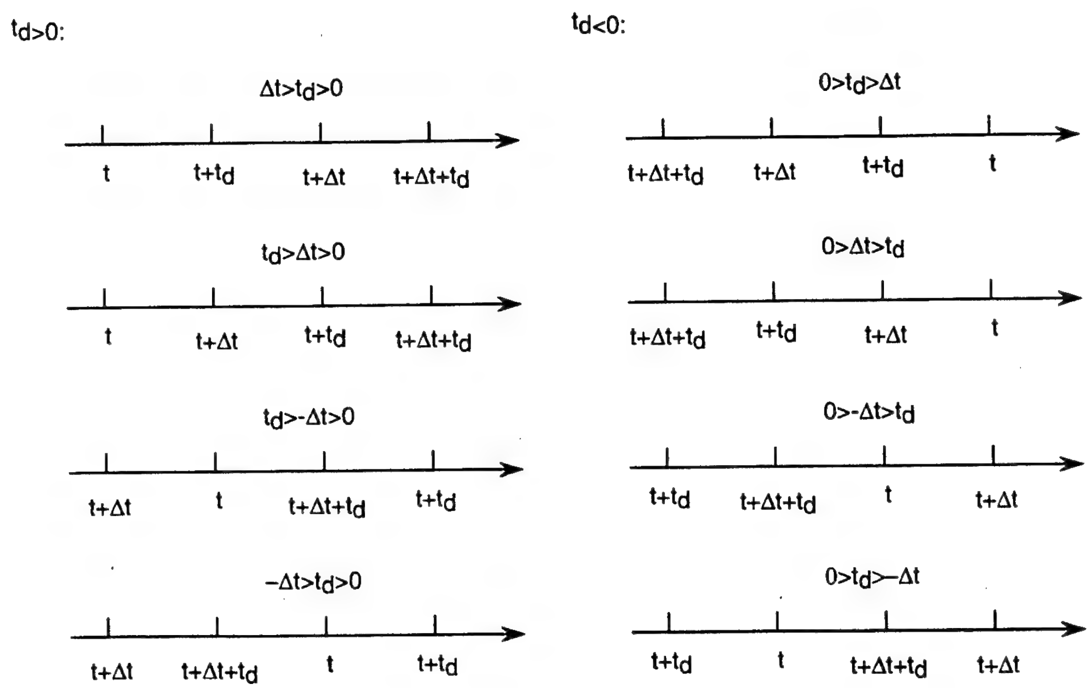

Fig. 2. Eight possible time sequences of $t + \Delta t + t_d$, $t + \Delta t$, $t + t_d$, and t.

$$R(0, \Delta t) = \begin{cases} I_1(0)I_2(0)\exp(-\delta\omega|t_d|) & |t_d| \le |\Delta t| \\ I_1(0)I_2(0)\exp(-\delta\omega|\Delta t|) & |t_d| > |\Delta t| \end{cases} \quad (29)$$

By use of Eqs. (17), (20), (24), (25), and (29), the signal intensity and the mutual coherence of the two waves can in general be obtained numerically.

B. Discussion

In this subsection we discuss some general characteristics and some special cases that can be solved analytically. We first describe an interesting relationship between the signal intensity and the mutual coherence of the two waves.

In general, the signal wave and the pump wave are partially coherent at the incidence plane. We can write the signal wave amplitude at the incidence plane as the sum of two terms, i.e.,

$$E_1(0, t) = E_{1c}(0, t) + E_{1i}(0, t), \quad (30)$$

where the term $E_{1c}(0, t)$ is fully coherent with the pump wave at the incidence plane and the term $E_{1i}(0, t)$ is totally incoherent with the pump wave at the incidence plane. This partition is unique. The two terms can be written as

$$E_{1c}(0, t) = \frac{\langle F(0, t)\rangle}{\sqrt{I_2(0)}} E_2(0, t), \quad (31)$$

$$E_{1i}(0, t) = E_1(0, t) - \frac{\langle F(0, t)\rangle}{\sqrt{I_2(0)}} E_2(0, t). \quad (32)$$

With Eqs. (30)–(32), we have separated the signal wave, before it enters the photorefractive medium, into two independent wave components. One is fully coherent with the pump wave and the other is totally incoherent with the pump wave. According to Eqs. (7)–(9), the coupled-wave equations are linear equations of the signal-wave amplitude. Therefore these two signal-wave components will also propagate through the photorefractive medium coupling with the pump wave independently. The amplitudes of two signal-wave components throughout the photorefractive medium are denoted $E_{1c}(z, t)$ and $E_{1i}(z, t)$, respectively. The intensities of the two signal-wave components are denoted $I_{1c}(z)$ and $I_{1i}(z)$, respectively. The autocorrelation function of the instantaneous mutual coherence of the two signal-wave components with the pump wave are denoted $R_c(z, \Delta t)$ and $R_i(z, \Delta t)$, respectively. Using Eqs. (31) and (32), we obtain

$$I_1(0) = I_{1c}(0) + I_{1i}(0), \quad (33)$$

$$R(0, \Delta t) = R_c(0, \Delta t) + R_i(0, \Delta t). \quad (34)$$

Note that the ensemble averages of the cross-product terms in Eqs. (33) and (34) are all zero. According to their definitions, $R_c(0, \Delta t)$ is a constant of the time delay Δt, whereas $R_i(0, \Delta t)$ approaches zero as the time delay Δt goes to infinity. Thus, given the function $R(0, \Delta t)$, we can obtain $R_c(0, \Delta t)$ and $R_i(0, \Delta t)$ as

$$R_c(0, \Delta t) = R(0, \infty), \quad (35)$$

$$R_i(0, \Delta t) = R(0, \Delta t) - R(0, \infty). \quad (36)$$

The propagation of the signal wave component that is fully coherent with the pump wave at the incidence plane can be solved explicitly. The boundary condition is given by $R_c(0, \Delta t) = I_{lc}(0)I_2(0)$. It is a constant of the time delay Δt, and its Fourier transform is a delta function. Using Eqs. (24) and (25), we obtain

$$R_c(z, \Delta t) = I_{1c}(0)I_2(0)\exp(\gamma z). \quad (37)$$

Note that the function $R_c(z, \Delta t)$ is a constant of the time delay Δt at any position z. The signal-wave intensity can thus be expressed as

$$I_{1c}(z) = I_{1c}(0)\exp(\gamma z). \quad (38)$$

With Eqs. (37) and (38), we can prove that the absolute value of the normalized mutual coherence of the two waves is 1 throughout the photorefractive medium. The signal-wave amplitude can be written as $E_{1c}(z, t) = E_{1c}(0, t - z/v)\exp(\gamma z/2)$. The signal intensity gain is the same as in the case in which the two waves are monochromatic waves. The normalized spectrum of the signal wave does not change in the two-wave mixing process and remains the same as the pump wave. Note that the propagation of the signal-wave component that is coherent with the pump wave is independent of the photorefractive relaxation time τ when the steady state is reached.

The above analysis shows that the signal-wave component that is totally coherent with the pump wave at the incidence plane remains totally coherent with the pump wave throughout the photorefractive medium. In addition, according to Eq. (18), the signal-wave component that is totally incoherent with the pump wave at the incidence plane remains totally incoherent with the pump wave throughout the photorefractive medium. Therefore, Eqs. (31)–(36) apply not only at the incidence plane but also throughout the photorefractive medium.

Since the propagations of the two signal-wave components are independent of each other and since the propagation of the signal-wave component that is totally coherent with the pump wave can be easily determined, we now consider the propagation of the signal wave component that is totally incoherent with the pump wave.

Without loss of generality, we consider the boundary condition given by Eq. (29). We assume that the time delay t_d between the signal wave and the pump wave is much larger than the coherence length of the source laser wave so that the two waves are totally incoherent. Under this assumption, the boundary condition [Eq. (29)] can be simplified as

$$R(0, \Delta t) = I_1(0)I_2(0)\exp(-\delta\omega|\Delta t|), \quad (39)$$

and its Fourier transform is

$$\tilde{R}(0, \xi) = I_1(0)I_2(0)\,\frac{1}{\pi}\,\frac{\delta\omega}{\delta\omega^2 + \xi^2}. \quad (40)$$

The function $\tilde{R}(0, \xi)$ has a Lorentzian shape. Its maximum is $\tilde{R}(0, 0) = I_l(0)I_2(0)/(\pi\delta\omega)$. Its FWHM is $2\delta\omega$, i.e., twice the bandwidth of the source laser wave. Generally speaking, when the signal wave and the pump wave are incoherent, the boundary condition $\tilde{R}(0, \xi)$ is a function of ξ with finite height and finite width; when the

signal wave and the pump wave are coherent, the boundary condition $\tilde{R}(0, \xi)$ is a delta function of ξ.

We now describe some general characteristics of the case in which the two waves are totally incoherent. Look at the function $\exp[\gamma z/(1 + \xi^2\tau^2)]$ in Eq. (25). It is an increasing function of γz. According to Eqs. (24) and (25), both the function $\tilde{R}(z, \xi)$ and the function $\tilde{R}(z, \Delta t)$ are increasing functions of γz. Thus, according to Eq. (20), the signal intensity will be amplified even if the two waves are incoherent. The function $\exp[\gamma z/(1 + \xi^2\tau^2)]$ is a decreasing function of the photorefractive relaxation time τ. According to Eqs. (20), (24), and (25), the signal intensity gain decreases as τ increases. The function $\exp[\gamma z/(1 + \xi^2\tau^2)]$ is also a decreasing function of $|\xi|$. Its FWHM with respect to the argument ξ is a decreasing function of γz. When $\gamma z \gg \ln 2$, the width is approximately $[\ln 2/(\gamma z)]^{1/2}/\tau$. According to Eqs. (24) and (25), the FWHM of the function $R(z, \Delta t)$ with respect to the argument Δt increases as γz increases. In other words, the correlation time of the instantaneous mutual coherence of the two waves increases as the signal wave becomes amplified.

We further consider some special cases that can be solved analytically.

First, we consider the case in which the photorefractive grating relaxation time constant is much shorter than the coherence time of the two interacting waves, i.e., $\delta\omega\tau \ll 1$. In this case, the width of the function $\tilde{R}(0, \xi)$ is much smaller than the width of the function $\exp[\gamma z/(1 + \xi^2\tau^2)]$, i.e., $\delta\omega\tau \ll [\ln 2/(4\gamma z)]^{1/2}$. Note that $[\ln 2/(4\gamma z)]^{1/2}$ is of the order of unity even for a relatively large value of γz. Therefore, the function $\exp[\gamma z/(1 + \xi^2\tau^2)]$ in Eqs. (24) and (25) can be replaced with $\exp(\gamma z)$, i.e., $\tau = 0$. This leads to

$$R(z, \Delta t) = R(0, \Delta t)\exp(\gamma z), \tag{41}$$

$$I_1(z) = I_1(0)\exp(\gamma z). \tag{42}$$

The signal intensity gain is the same as in the case of the two waves' being totally coherent. The function $R(z, \Delta t)$ retains its temporal profile while being amplified. In this case the photorefractive grating follows the fluctuation of the partially coherent optical waves instantaneously, i.e., $Q(z, t) = E_1(z, t')E_2{}^*(z, t')$ according to Eq. (7), and the signal wave amplitude can be directly obtained from Eq. (8) as $E_1(z, t) = E_1(0, t - z/v)\exp(\gamma z/2)$. Therefore the normalized spectrum of the signal wave does not change in the two-wave mixing process and does not have to be the same as the pump wave. This is in agreement with the fast response of the material. Since the signal intensity gain decreases as the photorefractive relaxation time τ increases when the two waves are totally incoherent, the signal intensity gain when the two waves are totally incoherent will in general be smaller than when the two waves are totally coherent and the photorefractive relaxation time τ is not negligible.

Second, we consider the case in which the photorefractive grating relaxation time constant is much longer than the coherence time of the two interacting waves, i.e., $\delta\omega\tau \gg 1$. In this case the width of the function $\tilde{R}(0, \xi)$ is much larger than the width of the function $\exp[\gamma z/(1 + \xi^2\tau^2)]$. As a result, the boundary condition $\tilde{R}(0, \xi)$ in Eq. (25) can be replaced with $\tilde{R}(0, 0) = I_1(0)I_2(0)/(\pi\delta\omega)$. When $\gamma z \ll 1$, the function $\exp[\gamma z/(1 + \xi^2\tau^2)]$ can be replaced with $1 + \gamma z/(1 + \xi^2\tau^2)$. This leads to

$$R(z, \Delta t) = R(0, \Delta t) + \frac{\gamma z}{\delta\omega\tau}\exp\left(-\frac{|\Delta t|}{\tau}\right), \tag{43}$$

$$I_1(z) = I_1(0)\left(1 + \frac{\gamma z}{\delta\omega\tau}\right). \tag{44}$$

The signal intensity gain coefficient is $\gamma z/(\delta\omega\tau)$ in the low-gain limit. The incremental change of the function $R(z, \Delta t)$, i.e., the second term on the right-hand side of Eq. (43), has a FWHM of $\tau \ln 2$, which is much larger than the FWHM of the function $R(0, \Delta t)$, i.e., $\ln 2/\delta\omega$. When $\gamma z \gg 1$, the function $\exp[\gamma z/(1 + \xi^2\tau^2)]$ can be replaced with $\exp[\gamma z(1 - \xi^2\tau^2)]$. This leads to

$$R(z, \Delta t) = \frac{1}{\delta\omega\tau}\frac{\exp(\gamma z)}{\sqrt{\pi\gamma z}}\exp\left(-\frac{|\Delta t|^2}{4\gamma z\tau^2}\right), \tag{45}$$

$$I(z) = \frac{I_1(0)}{\delta\omega\tau}\frac{\exp(\gamma z)}{\sqrt{\pi\gamma z}}. \tag{46}$$

In the high-gain limit, the signal intensity gain coefficient is close to γz, the same as in the case in which the two waves are totally coherent, whereas the signal intensity gain is at least $\delta\omega\tau$ times smaller than in the case in which the two waves are totally coherent. The temporal profile of the function $R(z, \Delta t)$ is Gaussian with a FWHM of $2\tau\sqrt{\gamma z}$. Codirectional two-wave mixing with partially coherent waves was studied by Bogodaev *et al.* under the assumption that the amplitude of the photorefractive grating is temporally stationary and proportional to the mutual coherence of the two interacting waves, i.e., $Q(z, t) = \langle F(z, t)\rangle$, in the case $\delta\omega\tau \gg 1$.[7] Under the undepleted pump approximation, their results show that the signal wave does not become amplified at all when it is incoherent with the pump wave, whereas the signal intensity and the mutual coherence of the two waves are the same as those given by Eqs. (18) and (38) when the two waves are totally coherent. The assumption of temporally stationary photorefractive grating is valid when the amplification of the signal intensity due to a signal-wave component incoherent with the pump wave is negligible compared with that due to a signal-wave component that is coherent with the pump wave.

C. Simulations

We now show some numerical results for the signal intensity gain using the boundary condition given by Eq. (29). In Fig. 3, we show the signal intensity gain as a function of the position z in the photorefractive medium for a given time delay t_d between the signal wave and the pump wave at the incidence plane and for five different values of the photorefractive grating relaxation time. The simulation parameters are interaction length $L = 1.0$ cm, index of refraction $n = 2.3$, photorefractive coupling coefficient $\gamma = 1.0\ \text{cm}^{-1}$, source laser wave bandwidth $\delta\omega = 2\pi\times 1.0 \times 10^6\ \text{s}^{-1}$, time delay $t_d = 0.3\ \mu$s, and five different values of the photorefractive grating relaxation time $\tau = 0.0$, 0.4, 1.0, 2.5, and ∞ μs. For the three finite cases, $\delta\omega\tau$ is of the order of unity. According to Fig. 3,

the signal intensity gain increases as functions of z in the photorefractive medium for a given photorefractive relaxation time τ and increases as the photorefractive relaxation time τ decreases. The signal intensity is the sum of the intensities of the signal-wave components that are coherent with the pump wave and of those incoherent with the pump wave. The intensity gain of the signal-wave component that is coherent with the pump wave is independent of the photorefractive relaxation time τ. Therefore, according to Fig. 3, the intensity gain of the signal-wave component that is incoherent with the pump wave increases as the photorefractive relaxation time τ decreases. When the photorefractive relaxation time τ is zero, the intensity gains of the two signal-wave components are equal. When the photorefractive relaxation time τ is not zero, the intensity gain of the signal wave component that is incoherent with the pump wave is smaller than that of the signal wave component that is coherent with the pump wave.

In Fig. 4 we show the signal intensity gain at the exit plane as a function of the time delay t_d between the two waves at the incidence plane for five different values of the photorefractive grating relaxation time. The simulation parameters are the same as those of Fig. 3. Since the curves of the signal intensity gain versus the time delay t_d are symmetric with respect to the axis when the time delay t_d is zero, we show only the curves for the positive time delay t_d. We notice that the signal intensity gains are decreasing functions of the time delay t_d. For all cases, the signal intensity gains reach asymptotic values when the time delay t_d is a couple of the coherence time of the source laser wave or larger. When $\delta\omega\tau$ is much larger than 1, there is no coupling between the two waves if the two waves are incoherent at the incidence plane owing to a large time delay (e.g., $t_d > 1\ \mu s$ in Fig. 4). When $\delta\omega\tau$ is of the order of unity or less, there is always coupling between the two waves, even if the two waves are completely incoherent at the incidence plane owing to a large time delay (e.g., $t_d > 1\ \mu s$ in Fig. 4). Also according to Fig. 4, the signal intensity gain increases as the photorefractive grating relaxation time decreases. The increase of the signal intensity gain with the decrease of the photorefractive grating relaxation

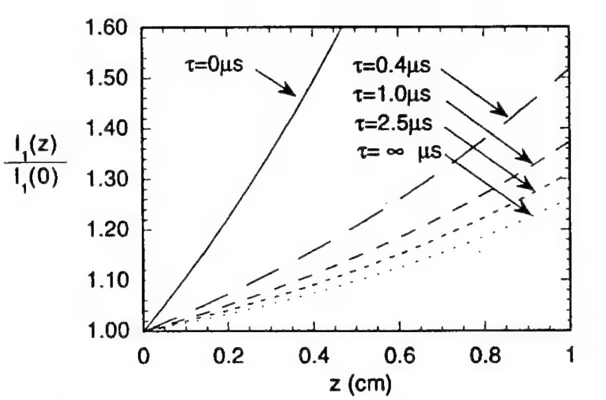

Fig. 3. Signal intensity gain as a function of the position in the photorefractive medium for a given time delay t_d between the signal wave and the pump wave at the incidence plane and for five different values of the photorefractive grating relaxation time.

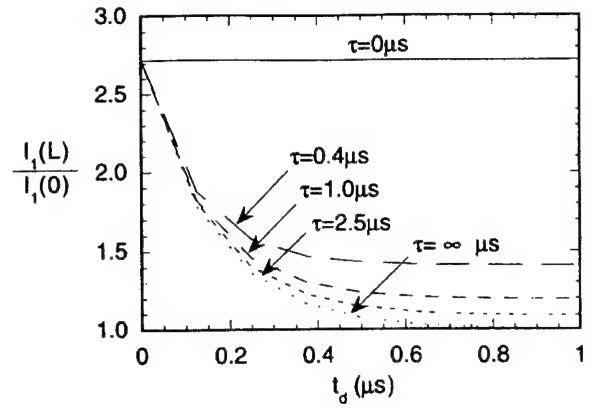

Fig. 4. Signal intensity gain at the exit plane as a function of the time delay t_d between the two waves at the incidence plane for five different values of the photorefractive grating relaxation time.

time becomes more significant when the time delay t_d is a couple of the coherence time of the source laser wave or larger. The dependence of the signal intensity gain on the time delay t_d between the two waves at the incidence plane can be easily understood as follows. Increasing the time delay t_d between the two waves increases the relative weight of the signal-wave component, which is incoherent with the pump wave. This explains the decrease of the signal intensity gain. When the time delay t_d is a couple of the coherence time of the source laser wave, the signal wave becomes completely incoherent with the pump wave and the signal intensity gain reaches its asymptotic value.

In this section the stochastic coupled-wave equations are reduced to nonstochatic differential equations, from which the signal intensity and the mutual coherence of the two waves can be obtained. The autocorrelation function of the instantaneous mutual coherence of the two waves at the incidence plane is found to be the necessary and sufficient boundary condition to determine the evolution of the signal intensity through the two-wave mixing process. Our results indicate that both the signal-wave component that is coherent with the pump wave and the signal-wave component that is incoherent with the pump can become amplified while maintaining their respective mutual coherence properties with the pump wave. The intensity gain of the signal-wave component that is incoherent with the pump wave depends on the photorefractive relaxation time constant, whereas the signal-wave component that is coherent with the pump wave does not. The effect of linear absorption can be easily taken into account.

4. SIGNAL SPECTRUM

In this section we solve for the signal spectrum. The stochastic coupled wave equations are rewritten into a series form, from which an asymptotic solution of the signal-wave spectrum can be obtained.

Since the signal-wave amplitude $E_1(z, t)$ is a stationary random process, the signal spectrum denoted $E_{11}(z, \omega)$ can be mathematically defined as

$$E_{11}(z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \langle E_1(z, t)E_1{}^*(z, t + \Delta t)\rangle \times \exp[-i(\omega - \omega_0)\Delta t]\mathrm{d}\Delta t, \tag{47}$$

where $\langle E_1(z, t)E_1{}^*(z, t + \Delta t)\rangle$ as a function of Δt is the autocorrelation function of the signal wave amplitude $E_1(z, t)$. According to the Wiener–Khintchine theorem, $E_{11}(z, \omega)d\omega$ is the average signal intensity that is due to frequency components between ω and $\omega + d\omega$. Since the signal-wave spectrum and the signal-wave autocorrelation function are Fourier transform pairs, we can obtain the signal spectrum by solving for the signal-wave autocorrelation function $\langle E_1(z, t)E_1{}^*(z, t + \Delta t)\rangle$.

The coupled-wave equations (7)–(9) can be expanded into a series of equations. First, we write the signal-wave amplitude $E_1(z, t)$ into the summation of a series as

$$E_1(z, t) = E_{10}(z, t) + E_{11}(z, t) + E_{12}(z, t) + \dots . \tag{48}$$

Each summation term in Eq. (48) is determined by the boundary condition and the series expansion of the coupled wave equations (7)–(9) as will be shown by Eqs. (50)–(54). We then write the grating amplitude $Q(z, t)$ into the summation of a series as

$$Q(z, t) = Q_{10}(z, t) + Q_{11}(z, t) + Q_{12}(z, t) + \dots , \tag{49}$$

where $Q_{1j}(z, t)$ is the component due to $E_{1j}(z, t)$ and the pump wave $E_2(z, t)$. We do not expand the pump-wave amplitude $E_2(z, t)$, which is a constant in our nondepleted pump approximation. Each component of the grating amplitude in Eq. (49) is determined by

$$\tau \frac{\partial Q_{1j}(z, t)}{\partial t} + Q_{1j}(z, t) = E_{1j}(z, t)E_2{}^*(z, t), \tag{50}$$

where the index $j = 0, 1, 2, \dots$. With each of the components determined this way, the grating amplitude will automatically satisfy Eq. (6) or Eq. (7). Physically, this means that the jth-order term of the grating amplitude is generated by the jth-order term of the signal wave and the pump wave. The boundary conditions at the incidence plane for the components of the signal-wave amplitude in Eq. (48) are given by

$$E_{10}(0, t) = E_1(0, t), \tag{51}$$

$$E_{1j}(0, t) = 0, \tag{52}$$

where $j = 1, 2, \dots$. In this way the boundary conditions at the incidence plane for the signal-wave amplitude are also automatically satisfied. The zeroth-order term of the signal wave is assumed to pass through the photorefractive medium without being affected by the photorefractive grating, i.e.,

$$\frac{\partial E_{10}(z, t)}{\partial z} + \frac{1}{V}\frac{\partial E_{10}(z, t)}{\partial t} = 0. \tag{53}$$

For $j > 0$, the jth order term of the signal wave is assumed to be generated by the pump wave reflection from the $(j - 1)$th order term of the photorefractive grating, i.e.,

$$\frac{\partial E_{1j}(z, t)}{\partial z} + \frac{1}{v}\frac{\partial E_{1j}(z, t)}{\partial t} = \frac{\gamma}{2}\frac{Q_{1(j-1)}(z, t)}{I_2(0)} E_2(z, t). \tag{54}$$

By summing both sides of Eq. (54) over $j = 0, 1, 2, \dots$, we obtain Eq. (8). Therefore Eqs. (48)–(54) are the series expansion form of the coupled-wave equations.

Using Eq. (48), we can expand the signal-wave autocorrelation function $\langle E_1(z, t)E_1{}^*(z, t + \Delta t)\rangle$ into a series summation as

$$\begin{aligned}\langle E_1(z, t)E_1{}^*(z, t + \Delta t)\rangle &= \langle E_{10}(z, t)E_{10}{}^*(z, t + \Delta t)\rangle \\ &\quad + [\langle E_{10}(z, t)E_{11}{}^*(z, t + \Delta t)\rangle \\ &\quad + \langle E_{11}(z, t)E_{10}{}^*(z, t + \Delta t)\rangle] + \dots .\end{aligned} \tag{55}$$

For example, the single product term $[E_{1m} \times (z, t)E_{1n}{}^*(z, t + \Delta t)]$ belongs to the $(m + n)$th order term of the autocorrelation function $\langle E_1(z, t) \times E_1{}^*(z, t + \Delta t)\rangle$. The product terms in Eq. (55) can be regrouped so that the autocorrelation function $\langle E_1(z, t)E_1{}^*(z, t + \Delta t)\rangle$ is expanded into a Taylor's series of γz.

The zeroth-order term of the Taylor series is

$$\langle E_{10}(z, t)E_{10}{}^*(z, t + \Delta t)\rangle = I_1(0)\exp\left(-\frac{1}{2}\Delta\omega|\Delta t|\right). \tag{56}$$

Equation (56) reflects that the source laser wave has been assumed to have a Lorentzian line shape. The first-order term of the Taylor series contains two single-product terms. These terms can be obtained from

$$\begin{aligned}\langle E_{11}(z, t)E_{10}{}^*(z, t + \Delta t)\rangle &= \frac{\gamma z}{2}\frac{1}{I_2(0)\tau}\int_{-\infty}^{0} \langle E_{10}(z, t + \Delta t')E_2{}^*(z, t + \Delta t') \\ &\quad \times E_2(z, t)E_{10}{}^*(z, t + \Delta t)\rangle \exp\left(\frac{\Delta t'}{\tau}\right)\mathrm{d}\Delta t',\end{aligned} \tag{57}$$

$$\langle E_{10}(z, t)E_{11}{}^*(z, t + \Delta t)\rangle = \langle E_{11}(z, t)E_{10}{}^*(z, t - \Delta t)\rangle^*. \tag{58}$$

Using the boundary condition given by Eq. (52), we can show that both sides of Eq. (57) are zero at the incidence plane $z = 0$. Using the series expansion form of the coupled-wave equations, we can show that the first derivatives over the position z of both sides of Eq. (57) are equal. This proves the validity of Eq. (57). According to Eq. (57), the coefficient of the first-order term of the Taylor series is related to a one-layer integration of a fourth-order coherence function of the two wave amplitudes at the incidence plane. When the source laser wave is assumed to have only phase noise, the fourth-order coherence function in Eq. (57) can be obtained in the same way as we obtain the fourth-order coherence function $R(0, \Delta t)$ given in Eq. (29). Its general expression is quite complicated. When the time delay t_d between the signal wave and the pump wave is further assumed to be much larger

than the coherence time of the source laser wave, the fourth-order coherence function in Eq. (57) can be expressed as

$$\langle E_{10}(z, t + \Delta t')E_2^*(z, t + \Delta t')E_2(z, t)E_{10}^*(z, t + \Delta t)\rangle$$
$$= I_1(0)I_2(0)\exp\left[-\frac{1}{2}\,\delta\omega(|\Delta t' - \Delta t| + |\Delta t'|)\right]. \quad (59)$$

Using Eq. (59), we obtain

$$\langle E_{10}(z, t)E_{11}^*(z, t + \Delta t)\rangle + \langle E_{11}(z, t)E_{10}^*(z, t + \Delta t)\rangle$$
$$= I_1(0)\exp\left(-\frac{1}{2}\,\delta\omega|\Delta t|\right)\frac{\gamma z}{\delta\omega\tau + 1}$$
$$\times\left\{1 + \frac{\delta\omega\tau}{2}\left[1 - \exp\left(-\frac{|\Delta t|}{\tau}\right)\right]\right\}. \quad (60)$$

In the case $\delta\omega\tau \ll 1$, Eq. (60) can be simplified as

$$\langle E_{10}(z, t)E_{11}^*(z, t + \Delta t)\rangle + \langle E_{11}(z, t)E_{10}^*(z, t + \Delta t)\rangle$$
$$= I_1(0)\exp\left(-\frac{1}{2}\,\delta\omega|\Delta t|\right)\gamma z. \quad (61)$$

In the case $\delta\omega\tau \gg 1$, Eq. (60) can be simplified as

$$\langle E_{10}(z, t)E_{11}^*(z, t + \Delta t)\rangle + \langle E_{11}(z, t)E_{10}^*(z, t + \Delta t)\rangle$$
$$= I_1(0)\exp\left(-\frac{1}{2}\,\delta\omega|\Delta t|\right)\frac{\gamma z}{\delta\omega\tau}. \quad (62)$$

According to Eqs. (56), (61), and (62), when the signal wave and the pump wave are independent at the incidence plane and the gain–length product γz is small compared with unity, the normalized spectrum of the signal wave does not change in the cases $\delta\omega\tau \ll 1$ and $\delta\omega\tau \gg 1$.

In general, there are $j + 1$ single product terms in the jth-order term of the Taylor series. The coefficient of the jth-order term of the Taylor series is related to j-level integrations of $(j + 1)$th-order coherence functions of the two wave amplitudes at the incidence plane. The coherence functions of the two wave amplitudes at the incidence plane can be obtained from the boundary conditions in the same way as we obtain the fourth-order coherence function $R(0, \Delta t)$ given in Eq. (29). As we have shown, the signal-wave intensity increases as $\exp(\gamma z)$. Since the signal-wave intensity is equal to the autocorrelation function $\langle E_1(z, t)E_1^*(z, t + \Delta t)\rangle$ at the time delay $\Delta t = 0$, we expect the correlation function to increase as $\exp(\gamma z)$ for the time delay $\Delta t \neq 0$ as well. Therefore the number of orders in the Taylor's series that is needed to achieve a certain accuracy increases quickly as γz increases. The series expansion method works most efficiently when γz is close to 1 or smaller. Fortunately, γz is usually small for high-speed photorefractive materials.

For example, when γz is small compared with unity, only the first two orders of the Taylor's series need to be taken into account. In Fig. 5 we show the first two orders of the Taylor's series separately as a function of the time delay Δt. Both functions are normalized by their respective maximum values. The time delay t_d between the two waves at the incidence plane is assumed to be much larger than the coherence time of the source laser wave. In this particular case, according to Eqs. (55) and (59), the

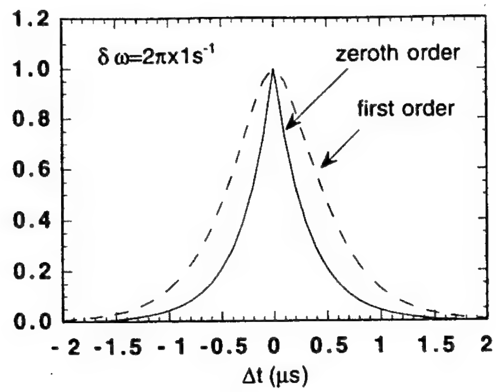

Fig. 5. First two orders of the Taylor's series of the autocorrelation function $\langle E_1(z, t)E_1^*(z, t + \Delta t)\rangle$ as a function of the time delay Δt.

maximum value of the zeroth-order term is $I_1(0)$, and the maximum value of the first-order term is $\gamma z I_1(0)/(\delta\omega\tau + 1)$. The bandwidth of the source laser wave is assumed to be 1 MHz. The photorefractive relaxation time is assumed to be 1 μs. According to Fig. 5, the widths of the first two orders of the autocorrelation function are not equal. The bandwidth of the signal wave will become narrower as the signal wave propagates through the photorefractive medium.

In this section we have presented an asymptotic solution of the signal spectrum. The solution is relatively simple in the low-gain limit. When the coherence time of the source laser wave is comparable with the photorefractive relaxation time, the signal wave spectrum will become narrower as the signal wave becomes amplified. When the coherence time of the source laser wave is much longer or much shorter than the photorefractive relaxation time, the signal wave spectrum remains unchanged.

5. CONCLUSIONS

We have investigated codirectional two-wave mixing with partially coherent waves in high-speed photorefractive media. When the spectral bandwidth of the optical waves is close to the speed of the photorefractive media, i.e., $\delta\omega\tau$ is of the order of unity, the photoinduced index grating will fluctuate with the amplitudes of the optical waves. The stochastic coupled-wave equations are solved in the nondepleted pump region with the assumption that the pump wave has only phase noise. Results for the evolution of the signal intensity, the mutual coherence of the two waves, and the signal spectrum are obtained. We found that the two-wave mixing process in high-speed photorefractive media is significantly different from that in low-speed photorefractive media. In the low-speed case we need to know only the intensities and the mutual coherence of the two waves at the incidence plane as the boundary condition to determine the evolution of the signal wave intensity, whereas in the high-speed case we need to know the autocorrelation function of the instantaneous mutual coherence of the two waves at the incidence plane. In the low-speed case a partial coherence between the two waves at the incidence plane is necessary for two-wave mixing and signal-wave amplification to occur, whereas in the high-speed case two-wave

mixing and signal-wave amplification will occur even if the two optical waves are incoherent at the incidence plane. In the low-speed case the two-wave mixing is independent of the photorefractive relaxation time constant, whereas, in the high-speed case the signal intensity gain increases as the photorefractive grating relaxation time decreases when the two optical waves are not fully coherent at the incidence plane. The evolution of the signal spectrum is also much more complicated in the high-speed case than in the low-speed case. In general, the signal-wave bandwidth changes as a result of the two-waving process. The analysis in the high-speed case also provides a better understanding of the limitation of the assumptions previously made in analyzing the low-speed case.

In our analysis the photorefractive medium has been assumed to be of purely diffusive type, which means that the coupling constant γ is a real number. With the presence of the photovoltaic effect or external electric field in the photorefractive medium, the coupling constant γ will be a complex number. The latter case can be analyzed similarly to the former. But the mathematical treatments involved will be slightly more complicated. The method used in our analysis can also be used to study stimulated Raman scattering with broadband light.

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REFERENCES

1. See, for example, P. Yeh, *Introduction to Photorefractive Nonlinear Optics* (Wiley, New York, 1993).
2. See, for example, P. Gunter and J.-P. Huignard, eds., *Photorefractive Materials and Devices I and II*, Vols. 61 and 62 of Topics in Applied Physics (Springer-Verlag, Berlin, 1988, 1989).
3. V. Wang, "Nonlinear optical phase conjugation for laser systems," Opt. Eng. **17**, 267 (1978).
4. S. C. De La Cruz, S. MacCormack, J. Feinberg, Q. B. He, H. K. Liu, and P. Yeh, "Effect of beam coherence on mutually pumped phase conjugators," J. Opt. Soc. Am. B **12**, 1363 (1995).
5. Q. B. He and P. Yeh, "Photorefractive mutually pumped phase conjugation with partially coherent beams," Appl. Phys. B **60**, 47 (1995).
6. R. Hofmeister, A. Yariv, and S. Yagi, "Spectral response of fixed photorefractive grating interference filters," J. Opt. Soc. Am. A **11**, 1342 (1994).
7. N. V. Bogodaev, L. I. Ivleva, A. S. Korshunov, N. M. Polozkov, and V. V. Shkunov, "Increase of light-beam coherence by two-wave mixing in photorefractive crystals," J. Opt. Soc. Am. B **10**, 2287 (1993).
8. M. Cronin-Golomb, H. Kong, and W. Krolikoski, "Photorefractive two-beam coupling with light of partial spatiotemporal coherence," J. Opt. Soc. Am. B **9**, 1698 (1992).
9. M. Horowitz and B. Fisher, "Parametric scattering with constructive and destructive light patterns induced by two mutually incoherent beams in photorefractive crystals," Opt. Lett. **17**, 1082 (1992).
10. D. Dolfi, A. Delboulbe, and J. P. Huignard, "Forward mixing of two mutually incoherent beams in a photorefractive crystal," Electron. Lett. **29**, 450 (1993).
11. D. Statman and B. Liby, "Two-beam cross coupling from mutually incoherent lasers," J. Opt. Soc. Am. B **6**, 1884 (1989).
12. S. Brognot, M. Defour, and J. P. Huignard, "Photorefractive 2WM: complex amplitudes solutions in case of a weak signal beam," Opt. Commun. **134**, 599 (1997).
13. X. Yi, S. H. Lin, P. Yeh, and K. Y. Hsu, "Contradirectional two-wave mixing with partially coherent waves in photorefractive crystals," Opt. Lett. **21**, 1123 (1996).
14. X. Yi, S. H. Lin, P. Yeh, and K. Y. Hsu, "General solution of contradirectional two-wave mixing with partially coherent waves in photorefractive crystals," J. Opt. Soc. Am. B **14**, 1396 (1997).
15. M. Mitchell, "Self-trapping of partially spatially incoherent light," Phys. Rev. Lett. **77**, 490 (1996).
16. P. Yeh, "Fundamental limit of the speed of photorefractive effect and its impact on device applications and material research," Appl. Opt. **26**, 602 (1987).
17. See, for example, J. W. Goodman, *Statistical Optics* (Wiley, New York, 1985), Chap. 5.

General solution of contradirectional two-wave mixing with partially coherent waves in photorefractive crystals

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We investigate contradirectional two-wave mixing with partially coherent waves in photorefractive crystals. By use of a statistical theory on linear systems, a general formulation of the problem in the space and frequency domain is derived and implemented numerically. We obtain results on beam intensity and mutual coherence. The results on the enhancement of mutual coherence are compared with previous theoretical results on simpler cases and with experimental measurements. Excellent agreements are achieved. The results also indicate that the effective interaction length can be significantly longer than the coherence length of the waves. © 1997 Optical Society of America [S0740-3224(97)01906-1]

1. INTRODUCTION

Two-wave mixing in photorefractive crystals has been investigated extensively for many applications including image amplification, laser-beam cleanup, spatial light modulators, thresholding, and power-limiting devices.[1,2] Most of the theoretical works in this area are based on wave-mixing with mutually coherent waves.[1,2] However, in some applications, such as self-pumped and mutually pumped phase-conjugate mirrors[3–6] and photorefractive filters,[7] the effect of partial temporal coherence in a two-wave-mixing process cannot be ignored. Two-wave mixing with partially coherent waves has been studied previously for the case of transmission-grating interaction.[8] In the case of transmission-grating interaction the optical path difference between the two interacting waves remains approximately the same as the two waves propagating codirectionally through the photorefractive medium, especially when the incident angles of the two waves are close to each other.[9] In the case of reflection-grating interaction the optical path difference between the two interacting waves varies significantly as the two waves propagate contradirectionally through the photorefractive medium. Thus for the case of transmission-grating interaction, only one free variable for the position is needed to describe in a self-consistent way the second-order statistical properties of the two optical waves, i.e., their intensities and mutual coherence, while at least two free variables, one for the position and one for the optical path difference, will be needed for the case of reflection-grating interaction. Another difficulty in studying the reflection-grating interaction of partially coherent waves is to find a way to incorporate the complete boundary conditions into the theoretical formulation as a result of the two-point boundary-value problem. In such a problem a complete set of boundary conditions includes both the second-order self-statistical properties (e.g., self-coherence) of each wave at its entrance boundary and the second-order mutual statistical properties (e.g., mutual coherence) of the two waves at their respective entrance boundaries. In a recent work we provided a theoretical formulation of the problem in the space and time domain for the reflection-grating interaction in the nondepleted-pump regime.[9] By using the nondepleted-pump approximation, we reduced the two-point boundary-value problem to an initial value problem. In this paper we present a general formulation of the problem in the space and frequency domain based on the standard statistical theory on linear systems. The general formulation is also implemented numerically. Specifically, we investigate the signal-intensity gain and the mutual coherence in the contradirectional wave mixing of two partially coherent waves. Contrary to conventional belief, we discover that the effective interaction length (or grating length) can be significantly longer than the coherence length of the incident waves. The results are also compared with previous theoretical results on simpler cases and with experimental measurements.

2. THEORETICAL MODEL

Photorefractive two-wave mixing is a nonlinear optical process. Because of the mutual coherence of the two waves, a dynamic holographic grating is formed in the medium. Its position and index profile are nearly stationary under the condition of a cw illumination. Both waves are scattered into each other by the presence of this index grating. Scattering of partially coherent waves by a stationary grating can be modeled with a statistical theory on linear systems.[10] An iterative procedure can subsequently be devised to obtain the final pho-

torefractive grating profile from an initially arbitrary grating profile.

As is shown in Fig. 1, two counter propagating waves with partial coherence enter a photorefractive medium at $z = 0$ and $z = L$, respectively. The electric field in the photorefractive medium can be written as

$$\mathbf{E}(z, t) = \mathbf{E}_1(z, t)\exp(-i\omega_0 t + i\mathbf{k}_0 z) + \mathbf{E}_2(z, t)\exp(-i\omega_0 t - i\mathbf{k}_0 z), \quad (1)$$

where ω_0 is the center frequency of the two partially coherent waves, $\mathbf{k}_0 = n\omega_0/c$ is the corresponding wave vector, and n is the refractive index of the photorefractive medium. Reflecting the partial coherence, $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ are stationary random variables. They represent the random fluctuation of the amplitudes of the two waves. For the convenience of our later discussion we will now briefly describe some notations and definitions for the second-order statistical properties of the two optical waves. Let $\Gamma_{11}(z, \tau) \equiv \langle \mathbf{E}_1(z, t_1)\mathbf{E}_1{}^*(z, t_2)\rangle$ and $\Gamma_{22}(z, \tau) \equiv \langle \mathbf{E}_2(z, t_1)\mathbf{E}_2{}^*(z, t_2)\rangle$ denote the self-coherence functions of $\mathbf{E}_1(z, t_1)$ and $\mathbf{E}_1(z, t_1)$, respectively, and $\Gamma_{12}(z, \tau) \equiv \langle \mathbf{E}_1(z, t_1)\mathbf{E}_2{}^*(z, t_2)\rangle$ be the mutual-coherence function between $\mathbf{E}_1(z, t_1)$ and $\mathbf{E}_2(z, t_2)$, where $\tau = t_1 - t_2$ is the time delay and $\langle\,\rangle$ means ensemble average. Let $\mathbf{E}_{11}(z, \Delta\omega)$ and $\mathbf{E}_{22}(z, \Delta\omega)$ denote the self-spectral-density functions of $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$, respectively, and $\mathbf{E}_{12}(z_1, z_2, \Delta\omega)$ be the cross-spectral-density function between $\mathbf{E}_1(z_1, t)$ and $\mathbf{E}_2(z_2, t)$. The spectral-density functions and the corresponding coherence functions are Fourier-transform pairs, i.e.,

$$\Gamma_{11}(z, \tau) = \int \mathbf{E}_{11}(z, \Delta\omega)\exp(-i\Delta\omega\tau)\mathrm{d}\Delta\omega, \quad (2)$$

$$\Gamma_{22}(z, \tau) = \int \mathbf{E}_{22}(z, \Delta\omega)\exp(-i\Delta\omega\tau)\mathrm{d}\Delta\omega, \quad (3)$$

$$\Gamma_{12}(z, \tau) = \int \mathbf{E}_{12}(z, z, \Delta\omega)\exp(-i\Delta\omega\tau)\mathrm{d}\Delta\omega, \quad (4)$$

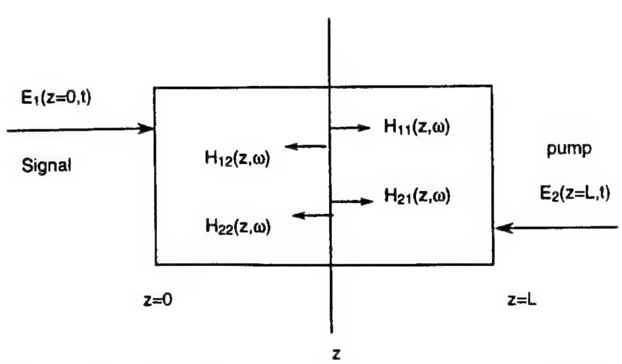


Fig. 1. Two-wave mixing in photorefractive crystals modeled as a linear system with the signal-wave entrance plane and the pump-wave entrance plane as two input planes and any arbitrary plane inbetween as the output plane.

where $\Delta\omega = \omega - \omega_0$, with ω being a general frequency component of the waves. With the above notations and relations the intensity of the two waves can be expressed as

$$I_1(z) \equiv \Gamma_{11}(z, 0) = \int \mathbf{E}_{11}(z, \Delta\omega)\mathrm{d}\Delta\omega, \quad (5)$$

$$I_2(z) \equiv \Gamma_{22}(z, 0) = \int \mathbf{E}_{22}(z, \Delta\omega)\mathrm{d}\Delta\omega, \quad (6)$$

and the mutual coherence of the two waves can be expressed as

$$\Gamma_{12}(z, 0) = \int \mathbf{E}_{12}(z, z, \Delta\omega)\mathrm{d}\Delta\omega. \quad (7)$$

With these equations the intensity of the waves as well as the mutual coherence can be obtained as soon as the spectral-density functions are obtained. In what follows we will derive the spectral-density functions by using the statistical approach.

Using the above notations and definitions, we now begin our discussion on the photorefractive interaction. Inside the photorefractive medium, a dynamic index grating is generated. It can be written as

$$\delta n = -i\frac{\gamma}{2}\frac{c}{\omega_0}\left[\frac{\mathbf{Q}(z, t)}{I_0(z)}\exp(2i\mathbf{k}_0 z) + \text{c.c.}\right], \quad (8)$$

where $\mathbf{Q}(z, t)$ is a measure of the index grating, γ is the intensity coupling coefficient, and $I_0(z) = I_1(z) + I_2(z)$ is the total intensity at position z. For the purpose of our discussion we will call $\mathbf{E}_1$ the signal wave and $\mathbf{E}_2$ the pump wave. Thus for a photorefractive grating with a positive γ, the signal wave $\mathbf{E}_1$ can be amplified. If the photorefractive effect is based purely on carrier diffusion (e.g., $BaTiO_3$,) the dynamics of the index grating is described by the following equation:

$$\tau_{\mathrm{ph}}\frac{\partial\mathbf{Q}(z, t)}{\partial t} + \mathbf{Q}(z, t) = \mathbf{E}_1(z, t)E_2{}^*(z, t), \quad (9)$$

where τ_{ph} is the relaxation-time constant. By virtue of photoexcitations, photorefractive processes are usually slow at low intensities. It is reasonable to assume that the coherence time $\delta\omega^{-1}$ of the two partially coherent waves is much smaller than the relaxation time τ_{ph} of the photorefractive medium, i.e., $\delta\omega\tau_{\mathrm{ph}} \gg 1$. Since $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ are stationary random variables, we can make the following approximation (see Appendix A)[8]:

$$\mathbf{Q}(z, t) \cong \langle\mathbf{Q}(z, t)\rangle = \Gamma_{12}(z, 0). \quad (10)$$

In other words, two partially coherent waves with their complex amplitudes fluctuating randomly with time can actually write a stationary grating in a photorefractive medium under the appropriate conditions. For simplicity we will denote $\mathbf{Q}(z, t)$ with $\mathbf{Q}(z)$ from now on. Note that $\mathbf{Q}(z)$ is also the mutual coherence of the two waves at position z. We note that the approximation is also valid when $\delta\omega\tau_{\mathrm{ph}} \ll 1$.

Given arbitrary functions of $\mathbf{Q}(z)$ and $I_0(z)$, Eq. (8) yields an index grating. The propagation of a monochromatic wave through such an index grating can be described by the coupled wave equations. The coupled monochromatic waves can be written as

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_1(z, \omega)\exp(-i\omega t + i\mathbf{k}z) + \tilde{\mathbf{E}}_2(z, \omega)\exp(-i\omega t - i\mathbf{k}z), \tag{11}$$

where $\tilde{\mathbf{E}}_1$ and $\tilde{\mathbf{E}}_2$ are the amplitudes of the monochromatic components, ω is the optical wave frequency, and $\mathbf{k} = n\omega/c$ is the optical wave vector. The coupled wave equations can be written as

$$\frac{\partial\tilde{\mathbf{E}}_1(z, \omega)}{\partial z} = \frac{\gamma}{2I_0(z)}\mathbf{Q}(z)\tilde{\mathbf{E}}_2(z, \omega)\exp(-2i\Delta\mathbf{k}z) - \frac{\alpha}{2}\tilde{\mathbf{E}}_1(z, \omega), \tag{12}$$

$$\frac{\partial\tilde{\mathbf{E}}_2(z, \omega)}{\partial z} = \frac{\gamma}{2I_0(z)}\mathbf{Q}^*(z)\tilde{\mathbf{E}}_1(z, \omega)\exp(2i\Delta\mathbf{k}z) + \frac{\alpha}{2}\tilde{\mathbf{E}}_2(z, \omega), \tag{13}$$

where $\Delta\mathbf{k} = \mathbf{k} - \mathbf{k}_0$ is the phase mismatch between the optical waves and the index grating and α is the intensity absorption coefficient. With sufficient boundary conditions, Eqs. (12) and (13) can be solved either analytically in some special cases or numerically in general. When the boundary conditions are $\tilde{\mathbf{E}}_1(z = 0, \omega) = 1$ and $\tilde{\mathbf{E}}_2(z = L, \omega) = 0$, the solutions (output) are denoted as $\tilde{\mathbf{E}}_1(z, \omega) = \mathbf{H}_{11}(z, \omega)$ and $\tilde{\mathbf{E}}_2(z, \omega) = \mathbf{H}_{12}(z, \omega)$. When the boundary conditions are $\tilde{\mathbf{E}}_1(z = 0, \omega) = 0$ and $\tilde{\mathbf{E}}_2(z = L, \omega) = 1$, the solutions (output) are denoted as $\tilde{\mathbf{E}}_1(z, \omega) = \mathbf{H}_{21}(z, \omega)$ and $\tilde{\mathbf{E}}_2(z, \omega) = \mathbf{H}_{22}(z, \omega)$. As a linear system, the general solutions are linear combinations of $\mathbf{H}_{11}$, $\mathbf{H}_{12}$, $\mathbf{H}_{21}$, and $\mathbf{H}_{22}$.

In each of the iterations an arbitrary stationary index grating can be considered as a linear system. Referring to Fig. 1, we consider a given stationary index grating in the photorefractive medium as a linear system with the optical waves at the boundary planes $z = 0$ and $z = L$ as the input and the optical waves at an arbitrary plane z as the output. According to the theory on the statistical properties of linear systems, the second-order statistical properties of the optical waves at the output plane can be expressed in terms of the second-order statistical properties of the optical waves at the input planes and the frequency response of the linear system. To be specific, the input at the $z = 0$ plane is $\mathbf{E}_1(z = 0, t)$, the input at $z = L$ plane is $\mathbf{E}_2(z = L, t)$, and the outputs at the z plane are $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$. The frequency response of this linear system can be expressed in terms of the solutions of Eqs. (12) and (13). With the notations described in the previous paragraph, the frequency responses from the input $\mathbf{E}_1(z = 0, t)$ to the outputs $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ are

$$\mathbf{H}_{11}'(z, \omega) = \mathbf{H}_{11}(z, \omega)\exp(i\Delta\mathbf{k}z), \tag{14}$$

$$\mathbf{H}_{12}'(z, \omega) = \mathbf{H}_{12}(z, \omega)\exp(-i\Delta\mathbf{k}z), \tag{15}$$

respectively. Similarly, the frequency responses from the input $\mathbf{E}_2(z = L, t)$ to the outputs $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ are

$$\mathbf{H}_{21}'(z, \omega) = \mathbf{H}_{21}(z, \omega)\exp(i\Delta\mathbf{k}L)\exp(i\Delta\mathbf{k}z), \tag{16}$$

$$\mathbf{H}_{22}'(z, \omega) = \mathbf{H}_{22}(z, \omega)\exp(i\Delta\mathbf{k}L)\exp(-i\Delta\mathbf{k}z), \tag{17}$$

respectively. The additional phase terms in Eqs. (14)–(17) account for the difference $\Delta\mathbf{k} = \mathbf{k} - \mathbf{k}_0$ owing to a finite $\Delta\omega = \omega - \omega_0$. With these spectral-response functions we can express the spectral-density functions of the two outputs $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ in terms of the spectral-density functions of the two inputs $\mathbf{E}_1(z = 0, t)$ and $\mathbf{E}_2(z = L, t)$. Thus, according to their definitions and Eqs. (14)–(17), we obtain

$$\begin{aligned}\mathbf{E}_{11}(z, \Delta\omega) = {} & \mathbf{H}_{11}(z, \omega)\mathbf{H}_{11}^*(z, \omega)\mathbf{E}_{11}(z = 0, \Delta\omega) \\ & + \mathbf{H}_{21}(z, \omega)\mathbf{H}_{21}^*(z, \omega)\mathbf{E}_{22}(z = L, \Delta\omega) \\ & + \mathbf{H}_{11}(z, \omega)\mathbf{H}_{21}^*(z, \omega) \\ & \times \mathbf{E}_{12}(z = 0, z = L, \Delta\omega)\exp(-i\Delta\mathbf{k}L) \\ & + \mathbf{H}_{21}(z, \omega)\mathbf{H}_{11}^*(z, \omega) \\ & \times [\mathbf{E}_{12}(z = 0, z = L, \Delta\omega) \\ & \times \exp(-i\Delta\mathbf{k}L)]^*,\end{aligned} \tag{18}$$

$$\begin{aligned}\mathbf{E}_{22}(z, \Delta\omega) = {} & \mathbf{H}_{12}(z, \omega)\mathbf{H}_{12}^*(z, \omega)\mathbf{E}_{11}(z = 0, \Delta\omega) \\ & + \mathbf{H}_{22}(z, \omega)\mathbf{H}_{22}^*(z, \omega)\mathbf{E}_{22}(z = L, \Delta\omega) \\ & + \mathbf{H}_{12}(z, \omega)\mathbf{H}_{22}^*(z, \omega) \\ & \times \mathbf{E}_{12}(z = 0, z = L, \Delta\omega)\exp(-i\Delta\mathbf{k}L) \\ & + \mathbf{H}_{22}(z, \omega)\mathbf{H}_{12}^*(z, \omega) \\ & \times [\mathbf{E}_{12}(z = 0, z = L, \Delta\omega) \\ & \times \exp(-i\Delta\mathbf{k}L)]^*,\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbf{E}_{12}(z, z, \Delta\omega) = {} & \exp(i2\Delta\mathbf{k}z)\{\mathbf{H}_{11}(z, \omega)\mathbf{H}_{12}^*(z, \omega) \\ & \times \mathbf{E}_{11}(z = 0, \Delta\omega) \\ & + \mathbf{H}_{21}(z, \omega)\mathbf{H}_{22}^*(z, \omega) \\ & \times \mathbf{E}_{22}(z = L, \Delta\omega) \\ & + \mathbf{H}_{11}(z, \omega)\mathbf{H}_{22}^*(z, \omega) \\ & \times \mathbf{E}_{12}(z = 0, z = L, \Delta\omega) \\ & \times \exp(-i\Delta\mathbf{k}L) + \mathbf{H}_{21}(z, \omega)\mathbf{H}_{12}^*(z, \omega) \\ & \times [\mathbf{E}_{12}(z = 0, z = L, \Delta\omega) \\ & \times \exp(-i\Delta\mathbf{k}L)]^*\}.\end{aligned} \tag{20}$$

Note that the spectral-density functions $\mathbf{E}_{11}(z = 0, \Delta\omega)$, $\mathbf{E}_{22}(z = L, \Delta\omega)$, and $\mathbf{E}_{12}(z_1 = 0, z_2 = L, \Delta\omega)$ of the two inputs $\mathbf{E}_1(z = 0, t)$ and $\mathbf{E}_2(z = L, t)$ are given as the boundary conditions.

Two physical processes happen simultaneously during two-wave mixing in a photorefractive medium. First, the two optical waves propagate through the photorefractive medium while being scattered into each other by the index grating. Second, the scattered waves modify the index grating through the photorefractive effect until a steady state is reached. We have provided above a mathematical model that describes these two physical processes separately. A steady state of the two-wave mixing in the photorefractive medium is reached when the two optical waves scattered by the photorefractive grating can

exactly sustain the same photorefractive grating. A steady-state solution of the two-wave mixing in a photorefractive medium can thus be obtained by use of the mathematical model described above through an iterative procedure. The procedure is outlined as follows:

Step 1: Give an initial guess on the function $\mathbf{Q}(z)/I_0(z)$.

Step 2: Solve Eqs. (12) and (13) for the functions $\mathbf{H}_{ij}(z, \omega)$ $(i, j = 1, 2)$ with the function $\mathbf{Q}(z)/I_0(z)$ provided in the last step.

Step 3: Obtain the spectral-density functions $\mathbf{E}_{11}(z, \Delta\omega)$, $\mathbf{E}_{22}(z, \Delta\omega)$, and $\mathbf{E}_{12}(z, z, \Delta\omega)$ by use of Eqs. (18)–(20).

Step 4: Obtain $I_1(z)$, $I_2(z)$, $\mathbf{Q}(z)$, and $\mathbf{Q}(z)/I_0(z)$ by use of Eqs. (5), (6), (7), and (10).

Step 5: Compare the new version of the index grating $\mathbf{Q}(z)/I_0(z)$ and the previous version. If they are close within a certain accuracy requirement, the solution has been obtained. Otherwise, the iteration continues by use of the new version of the index grating.

To determine the boundary conditions, we assume that both the input optical waves are derived from the same laser source. If the source laser wave has a Gaussian line shape with a FWHM linewidth of $\delta\omega$, then the normalized spectral-density function of the source laser wave can be written as

$$\mathbf{E}_{ss}(\Delta\omega) = \frac{4(\pi \ln 2)^{1/2}}{\delta\omega} \exp\left\{-\left[2(\ln 2)^{1/2} \frac{\Delta\omega}{\delta\omega}\right]^2\right\}. \tag{21}$$

Taking β as the incident-intensity ratio $I_1(z = 0)/I_2(z = L)$ of the two optical waves at their respective entrance boundary planes, we obtain the boundary conditions as

$$\mathbf{E}_{11}(z = 0, \Delta\omega) = \beta \mathbf{E}_{ss}(\Delta\omega), \tag{22}$$

$$\mathbf{E}_{22}(z = 0, \Delta\omega) = \mathbf{E}_{ss}(\Delta\omega), \tag{23}$$

$$\mathbf{E}_{12}(z_1 = 0, z_2 = L, \Delta\omega) = \sqrt{\beta}\mathbf{E}_{ss}(\Delta\omega)\exp(-i\mathbf{k}_0 L) \times \exp(-i\omega t_d), \tag{24}$$

where t_d is the time delay between the optical waves when they reach their respective entrance planes. In deriving the above boundary conditions, we have assumed that the laser source has a Gaussian line shape. There is no loss of generality in this assumption. Similar results can be obtained with a different line shape.

This completes the general formulation to model contradirectional two-wave mixing in photorefractive crystals.

3. DISCUSSIONS AND SIMULATIONS

To clarify the complicated formulation described above, we consider some simple cases first before presenting the numerical and experimental results.

In the absence of coupling ($\gamma = 0$) in a lossless medium ($\alpha = 0$) with $\beta = 1$ the above formulation describes the interference of two counterpropagating optical waves in a dielectric medium. In this case it takes only one iteration to obtain the solution, which is

$$\mathbf{E}_{12}(z, z, \Delta\omega) = \mathbf{E}_{ss}(\Delta\omega)\exp(-i\mathbf{k}L) \times \exp(-i\omega t_d)\exp(2i\Delta\mathbf{k}z). \tag{25}$$

Fourier transforming the above equation over ω and taking into account the extra phase term $\exp(2i\mathbf{k}_0 z)$ resulted from the definition of $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ in Eq. (1), we obtain the mutual coherence of the two waves as a function of position z,

$$\Gamma_{12}(z, \tau) = \Gamma_s\left(\tau + t_d + \frac{nL - 2nz}{c}\right)\exp(-i\omega_0 t_d), \tag{26}$$

where $\Gamma_s(\tau)$ is the normalized self-coherence function of the source laser wave and n is the index of refraction of the medium. The result has been well established in the literature. This example can also help us see more clearly the subtle contribution of the phase terms in the above formulation. In our definition, $t_d = 0$ if $z = L/2$ is the plane of zero path difference.

When the coherence time $\delta\omega^{-1}$ is much larger than the time delay involved in the above formulation, i.e., $\delta\omega t_d \ll 1$ and $\delta\omega nL/c \ll 1$, the normalized spectral-density function of the source laser wave can be written as $\mathbf{E}_{ss}(\omega) = \delta(\omega - \omega_0)$. From the above formulation we can obtain the set of equations governing the intensities of the two optical waves,

$$\frac{d}{dz} I_1 = \gamma \frac{I_1 I_2}{I_1 + I_2} - \alpha I_1, \tag{27}$$

$$\frac{d}{dz} I_1 = \gamma \frac{I_1 I_2}{I_1 + I_2} + \alpha I_1. \tag{28}$$

These two equations are exactly the same as those obtained for two-wave mixing of monochromatic waves. We note that $\delta\omega t_d \ll 1$ is a condition for partially coherent waves to be treated as monochromatic waves in contradirectional two-wave mixing in a photorefractive medium. We also recall that the upper limit on the coherence time, i.e., $\delta\omega\tau_{ph} \gg 1$, still needs to be satisfied to reach Eqs. (27) and (28) from the above general formulation. This is the main difference between the results of Eqs. (27) and (28) in this paper and those obtained directly for monochromatic waves. Note that we did not use the iterative procedure in obtaining Eqs. (27) and (28). The complete boundary conditions for Eqs. (27) and (28) are simply $I_1(z = 0)$ and $I_2(z = L)$.

We now use the above general formulation to obtain the solution in the nondepleted-pump regime (for $\gamma \neq 0$), which has been obtained previously by a different method. Using Eqs. (12) and (13) and the following three definitions,

$$\mathbf{E}_{11}(z, \Delta\omega) = \langle \tilde{\mathbf{E}}_1(z, \omega)\tilde{\mathbf{E}}_1{}^*(z, \omega)\rangle, \tag{29}$$

$$\mathbf{E}_{22}(z, \Delta\omega) = \langle \tilde{\mathbf{E}}_2(z, \omega)\tilde{\mathbf{E}}_2{}^*(z, \omega)\rangle, \tag{30}$$

$$\mathbf{E}_{12}(z, z, \Delta\omega) = \langle \tilde{\mathbf{E}}_1(z, \omega)\tilde{\mathbf{E}}_2{}^*(z, \omega)\rangle\exp(2i\Delta\mathbf{k}z), \tag{31}$$

we can obtain a set of equations governing the propagation of the spectral-density functions as

$$\frac{\partial \mathbf{E}_{11}(z, \Delta\omega)}{\partial z} = \frac{\gamma}{2I_0(z)} [\mathbf{Q}(z)\mathbf{E}_{12}^*(z, z, \Delta\omega) + \mathbf{Q}^*(z)\mathbf{E}_{12}(z, z, \Delta\omega)] - \alpha\mathbf{E}_{11}(z, \Delta\omega), \quad (32)$$

$$\frac{\partial \mathbf{E}_{22}(z, \Delta\omega)}{\partial z} = \frac{\gamma}{2I_0(z)} [\mathbf{Q}(z)\mathbf{E}_{12}^*(z, z, \Delta\omega) + \mathbf{Q}^*(z)\mathbf{E}_{12}(z, z, \Delta\omega)] + \alpha\mathbf{E}_{22}(z, \Delta\omega), \quad (33)$$

$$\frac{\partial \mathbf{E}_{12}(z, z, \Delta\omega)}{\partial z} = 2i\Delta\mathbf{k}\mathbf{E}_{12}(z, z, \Delta\omega) + \frac{\gamma}{2I_0(z)}\mathbf{Q}(z) \times [\mathbf{E}_{11}(z, \Delta\omega) + \mathbf{E}_{22}(z, \Delta\omega)] - \alpha\mathbf{E}_{12}(z, z, \Delta\omega). \quad (34)$$

$\tilde{\mathbf{E}}_1(z, \omega)$ and $\tilde{\mathbf{E}}_2(z, \omega)$ in Eqs. (29)–(31) are related to the Fourier-transform coefficients of the two optical waves as defined in Eq. (11). Strictly speaking, a stationary random process cannot be Fourier transformed over the time variable t. However, we can truncate it into a finite duration T and Fourier transform the truncated process. After that, we can first obtain a set of equations similar to Eqs. (32)–(34) for the truncated process, then let T go to infinity to obtain Eqs. (32)–(34). This is a standard procedure in statistical optics.[10] Fourier transforming the above relation over $\Delta\omega$, we obtain a set of equations governing the propagation of the self and mutual coherence of the two optical waves as

$$\frac{\partial \Gamma_{12}(z, \tau)}{\partial z} = -\frac{2n}{c}\frac{\partial \Gamma_{12}(z, \tau)}{\partial \tau} + \frac{\gamma}{2}\frac{\Gamma_{12}(z, 0)}{I_1 + I_2} \times [\Gamma_{11}(z, \tau) + \Gamma_{22}(z, \tau)], \quad (35)$$

$$\frac{\partial \Gamma_{11}(z, \tau)}{\partial z} = \frac{\gamma}{2}\frac{\Gamma_{12}(z, 0)}{I_1 + I_2}\Gamma_{12}^*(z, -\tau) + \frac{\gamma}{2}\frac{\Gamma_{12}^*(z, 0)}{I_1 + I_2}\Gamma_{12}(z, \tau) - \alpha\Gamma_{11}(z, \tau), \quad (36)$$

$$\frac{\partial \Gamma_{22}(z, \tau)}{\partial z} = \frac{\gamma}{2}\frac{\Gamma_{12}(z, 0)}{I_1 + I_2}\Gamma_{12}^*(z, -\tau) + \frac{\gamma}{2}\frac{\Gamma_{12}^*(z, 0)}{I_1 + I_2}\Gamma_{12}(z, \tau) + \alpha\Gamma_{22}(z, \tau). \quad (37)$$

When there is no absorption in the photorefractive medium (i.e., $\alpha = 0$), we can obtain, from Eqs. (36) and (37),

$$\frac{\partial}{\partial z}[\Gamma_{11}(z, \tau) - \Gamma_{22}(z, \tau)] = 0. \quad (38)$$

Further, letting $\tau = 0$ in Eq. (38), we obtain

$$\frac{\partial}{\partial z}[I_1(z) - I_2(z)] = 0. \quad (39)$$

Equation (39) shows that, when there is no absorption in the photorefractive medium, the intensity difference between the signal wave and the pump wave is a constant of integration (conservation of power flow).

Note that, although Eqs. (35)–(37) are general and self-consistent, they contain only the mutual coherence of the two optical waves at the same locations. They are not compatible with the kind of boundary conditions as given by Eq. (24), which gives the mutual statistical properties of the two waves at two different points in space. Therefore they are useful only in the nondepleted-pump regime, where we can assume that the pump wave passes through the photorefractive medium without changing its statistical properties. Under this assumption we can obtain from Eqs. (22)–(24) the self- and mutual-coherence functions of the two waves at the signal-wave entrance boundary plane ($z = 0$) as

$$\Gamma_{12}(z = 0, \tau) \equiv \sqrt{\beta}\Gamma_{ss}(\tau + \delta t)\exp(-i\omega_0\delta t), \quad (40)$$

$$\Gamma_{11}(z = 0, \tau) \equiv \beta\Gamma_{ss}(\tau), \quad (41)$$

$$\Gamma_{22}(z = 0, \tau) \equiv \Gamma_{ss}(\tau), \quad (42)$$

where $\delta t = t_d + nL/c$ is the time delay between the two optical waves at the signal-wave entrance plane and

$$\Gamma_{ss}(\tau) = \exp\left\{-\left[\frac{\delta\omega\tau}{4(\ln 2)^{1/2}}\right]^2\right\} \quad (43)$$

is the coherence function of the source laser wave. We can use Eqs. (40)–(42) as the boundary conditions and integrate Eqs. (35)–(37).

Equations (35)–(37) are simple not only in the sense that they can be implemented easily numerically, but also that they can be used to obtain approximate analytical solutions to some special cases within the nondepleted regime. One such case is that the coherence length of the source laser wave is much longer than the two-wave-mixing interaction length and the coupling constant is large. This occurs when we use a multimode argon laser and a $KNbO_3$:Co crystal for the two-wave-mixing experiment. In this case we can neglect the term that contains the partial derivative on τ in Eq. (35) and reduce the set of partial differential equations, i.e., Eqs. (35)–(37), to a set of ordinary differential equations that can be solved analytically under the nondepleted-pump approximation. Remember that Eq. (35) is derived directly from Eq. (34). We can see from Eq. (34) that the approximation of neglecting the term that contains the partial derivative on τ in Eq. (35) implies that the wave-vector difference $\Delta\mathbf{k}$ of the different frequency components of the two partially coherent waves are negligible with respect to the thickness of the photorefractive medium (i.e., $\Delta\mathbf{k}L \ll 1$). Similar approximation has been made previously by Saxena *et al.* in the study of multiple-beam interaction by transmission gratings in the photorefractive media.[12,13] Therefore under the nondepleted-pump approximation and the approximation of $\Delta\mathbf{k}L \ll 1$ the signal-wave intensity gain and the normalized mutual coherence of the two waves can be obtained as

$$\frac{I_1(z)}{I_1(0)} \equiv \frac{\Gamma_{11}(z, 0)}{\Gamma_{11}(0, 0)} = \frac{\Gamma_{12}(0, 0)\Gamma_{12}^*(0, 0)}{\Gamma_{11}(0, 0)\Gamma_{22}(0, 0)} \times [\exp(\gamma z) - 1]\exp(-\alpha z) + \exp(-\alpha z), \quad (44)$$

$$\gamma_{12}(z) \equiv \frac{\Gamma_{12}(z, 0)}{[\Gamma_{11}(z, 0)\Gamma_{22}(z, 0)]^{1/2}} = \frac{1}{\left\{\dfrac{\Gamma_{12}^*(0, 0)}{\Gamma_{12}(0, 0)}[1 - \exp(-\gamma z)] + \dfrac{\Gamma_{11}(0, 0)\Gamma_{22}(0, 0)}{\Gamma_{12}^2(0, 0)}\exp(-\gamma z)\right\}^{1/2}}. \qquad (45)$$

In most photorefractive crystals with $\exp[(\gamma - \alpha)z] \gg 1$ the signal-intensity gain is affected primarily by $(\gamma - \alpha)$. We also note that the normalized mutual coherence of the two waves is affected only by the photorefractive coupling constant γ.

As a comparison of the general formulation, the simplified formulation and the approximate analytical solutions, we show in Figs. 2 and 3 the signal-intensity gain and the normalized mutual coherence of the two waves at the signal-wave exit plane ($z = L$) as a function of the optical path difference between the two optical waves at the signal-wave entrance plane ($z = 0$). The parameters are $\gamma = 3.0\ \text{cm}^{-1}$, $\alpha = 0.0\ \text{cm}^{-1}$, $n = 2.3$, $L = 0.72$ cm, and $\delta\omega = 2\pi \times 1.8$ GHz, and $\beta = 10^{-4}$. These are typical parameters in our experiment when we use a multimode argon laser and a $KNbO_3$:Co crystal to implement the two-wave-mixing experiment. In this case the coherence length of the source laser wave is much longer than the thickness of the photorefractive medium. The general formulation and the simplified formulation produce the same results within the numerical accuracy in the nondepleted regime. The results of these two formulations are represented by the dashed curves in Figs. 2 and 3. We notice that the approximate analytical solution retains the major characteristics of the exact solution and provides a good understanding of the interaction. The results of the approximate analytical solution are represented by the solid curves in the figures. Note that the curves obtained from the approximate analytical solution are symmetric about $z_0 = 0$ owing to the approximation of neglecting the partial derivative over τ in Eq. (35), while the curves obtained from the general formulation are shifted to the right side (see Figs. 2 and 3). Here, z_0 is defined as the path difference. The difference between the results of the approximate analytical solution and those of the general formulation will decrease as the ratio between the coherence length of two waves and the thickness of the photorefractive medium increases.

Now let us consider the case of the depleted pump. Numerical simulation with the general formulation is the only means to analyze this case. We first consider the case in which the coherence length of the source laser wave is finite but much longer than the thickness of the photorefractive medium. Again, we use the following set of parameters: $\gamma = 3.0\ \text{cm}^{-1}$, $\alpha = 0.0\ \text{cm}^{-1}$, $n = 2.3$, $L = 0.72$ cm, and $\delta\omega = 2\pi \times 1.8$ GHz. The solid curves in Fig. 4 show the signal intensity $I_1(z)$, the pump intensity $I_2(z)$, and the normalized mutual coherence $\gamma(z) = \Gamma_{12}(z, 0)/[\Gamma_{11}(z, 0)\Gamma_{22}(z, 0)]^{1/2}$ as functions of the position z in the photorefractive medium. The optical path difference between the signal wave and the pump wave at the signal-wave entrance boundary ($z = 0$) is chosen to be zero. The incidence intensity ratio β of the two waves is chosen to be one. We also show in Fig. 4 with dashed curves the signal intensity $I_1(z)$ and the pump intensity $I_2(z)$ for two-wave mixing with monochromatic waves for the purpose of comparison. We notice that the results of two-wave mixing with partially coherent waves are very close to those of two-wave mixing with

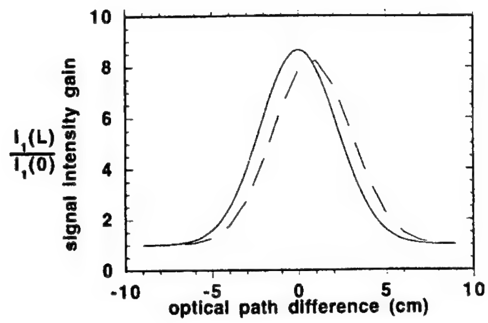


Fig. 2. Signal-wave intensity gain as a function of the optical-path difference at the signal-wave entrance plane in the nondepleted-pump regime. The dashed curve is the numerical solution. The solid curve is the approximate analytical solution.

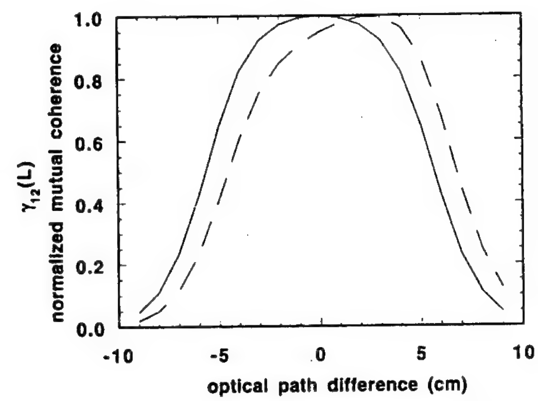


Fig. 3. Mutual coherence of the two waves at the signal-wave exit plane as a function of the optical path difference at the signal-wave entrance plane in the nondepleted-pump regime. The dashed curve is the numerical solution. The solid curve is the approximate analytical solution.

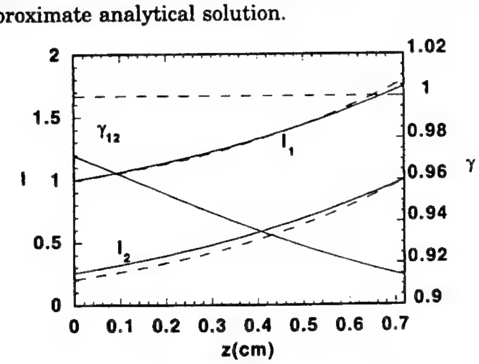
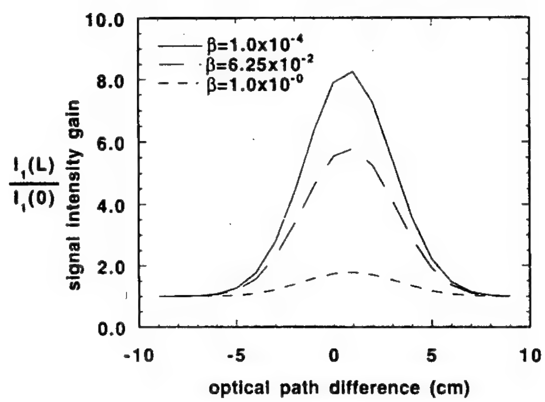
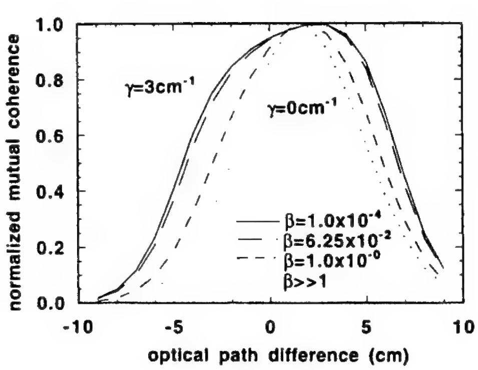


Fig. 4. Signal intensity, pump intensity, and the normalized mutual coherence as a function of position z in the photorefractive medium for partially coherent waves (solid curves) and monochromatic waves (dashed curves).

monochromatic waves when the optical path difference of the two waves is small compared with the coherence length of the source laser wave. The effect of partial coherence of the two waves in this case is significant only for a large optical path difference of the two waves. We show the signal-intensity gain in Fig. 5(a) and the mutual coherence between the signal wave and the pump wave at the pump-wave entrance plane ($z = L$) in Fig. 5(b) both as functions of the optical path difference between the two waves at the signal-wave entrance plane ($z = 0$) for various intensity ratios β. Figure 5(a) shows that the signal-intensity gain decreases as the optical path difference of the two waves increases until there is no coupling between the two waves when the optical path difference of the two waves exceeds the coherent length of the source laser wave. The same figure also shows that the signal-intensity gain increases as the intensity ratio β decreases until the signal-intensity gain saturates as the nondepleted-pump regime is reached. Figure 5(b) shows that the normalized mutual coherence of the two waves at the pump-wave entrance plane ($z = L$) decreases as the intensity ratio β of the two waves increases. When β is much larger than one, the signal wave will pass through the photorefractive medium with its statistical properties almost unchanged by coupling. In this limit the normalized mutual coherence of the two waves at the pump-wave entrance plane ($z = L$) with coupling will be the same as that for an arbitrary β but without coupling. We note that the normalized mutual coherence of the two waves at the pump-wave entrance plane ($z = L$) is enhanced by coupling. Figure 5(b) also shows that the normalized mutual coherence decreases quickly as the optical path difference gets close to and larger than the coherence length of the source laser wave.



(a)



(b)

Fig. 5. (a) Signal-intensity gain and (b) the normalized mutual coherence of the two waves at the pump-wave entrance plane ($z = L$) as functions of the optical path difference at the signal-wave entrance plane ($z = 0$) for a coupling constant $\gamma = 3\ \mathrm{cm}^{-1}$ and various intensity ratios between the signal wave and the pump wave. Note that the curve for a coupling constant $\gamma = 3\ \mathrm{cm}^{-1}$ and $\beta \gg 1$ is the same as that for a coupling constant $\gamma = 0\ \mathrm{cm}^{-1}$ and an arbitrary β.

Second, we consider the case that the coherence length of the source laser wave is shorter than the thickness of the photorefractive medium. The following parameters are chosen in the simulation: $\alpha = 0.0\ \mathrm{cm}^{-1}$, $n = 2.3$, $L = 2.0$ cm, $\delta\omega = 2\pi \times 18$ GHz, and $t_d = -nL/c$. $t_d = -nL/c$ implies that the optical path difference between the signal wave and the pump wave is zero at the signal-wave entrance plane ($z = 0$). Since the spectral line shape of the source laser wave is assumed to be Gaussian, the coherence length of the source laser wave inside the photorefractive medium is $L_c = 2\pi \times 0.664c/(n\,\delta\omega) = 0.48$ cm. In Fig. 6 we show the signal intensity $I_1(z)$ and the grating profile $\mathbf{Q}(z)/I_0(z)$ as functions of the position z inside the photorefractive medium for an incident intensity ratio $\beta = 1$ and a coupling constant $\gamma = 20\ \mathrm{cm}^{-1}$. We note that the length of the photorefractive grating is limited by the partial coherence of the two interacting waves and that the pump depletion is moderate even for a very large coupling constant. In Figs. 7(a) and 7(b) we show the signal intensity $I_1(z)$ and the grating profile $\mathbf{Q}(z)/I_0(z)$ as functions of the position z inside the photorefractive medium for an incident intensity ratio $\beta = 10^{-4}$ and coupling constants $\gamma = 10\ \mathrm{cm}^{-1}$ and $\gamma = 20\ \mathrm{cm}^{-1}$, respectively. We note that the length of the photorefractive grating is increased but still primarily limited by the partial coherence of the two interacting waves for a small incident intensity ratio and a small coupling constant. Figure 7(b) shows that the length of the photorefractive grating is no longer limited by the partial coherence of the two interacting waves for a small incident intensity ratio and a large coupling constant. The length of the photorefractive grating in this case is limited by the length of the photorefractive medium, which is much longer than the coherence length of the interacting waves. The photorefractive grating is a temporally stationary index grating. When the incident intensity ratio is small, the amplified signal wave at its exit plane ($z = L$) is primarily the incident pump wave reflected by the photorefractive grating, and the depleted pump wave at its exit plane ($z = 0$) is primarily the incident pump wave transmitted through the photorefractive grating. When the length of the photorefractive grating is comparable to or longer than the coherence length of the source laser wave inside the photorefractive medium, the output waves $\mathbf{E}_1(L)$ and $\mathbf{E}_2(0)$ may have spectra different from those of the input waves $\mathbf{E}_1(0)$ and $\mathbf{E}_2(L)$ owing to the presence of the photorefractive grating, which acts as a spectral filter. In Figs. 8(a) and 8(b) we show the normalized spectra of the amplified signal wave and the depleted pump wave at their respective exit planes, using the same sets of parameters as those used to obtain Figs. 7(a) and 7(b), respectively. We note that the amplified signal wave has a bandwidth narrower than that of the

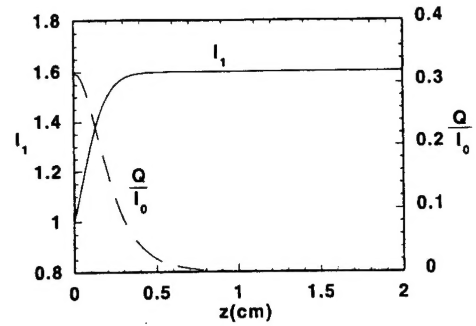


Fig. 6. Signal intensity $I_1(z)$ (solid curve) and the grating profile $\mathbf{Q}(z)/I_0(z)$ (dashed curve) as functions of the position z inside the photorefractive medium for an incident intensity ratio $\beta = 1$ and a coupling constant $\gamma = 20\ \mathrm{cm}^{-1}$.

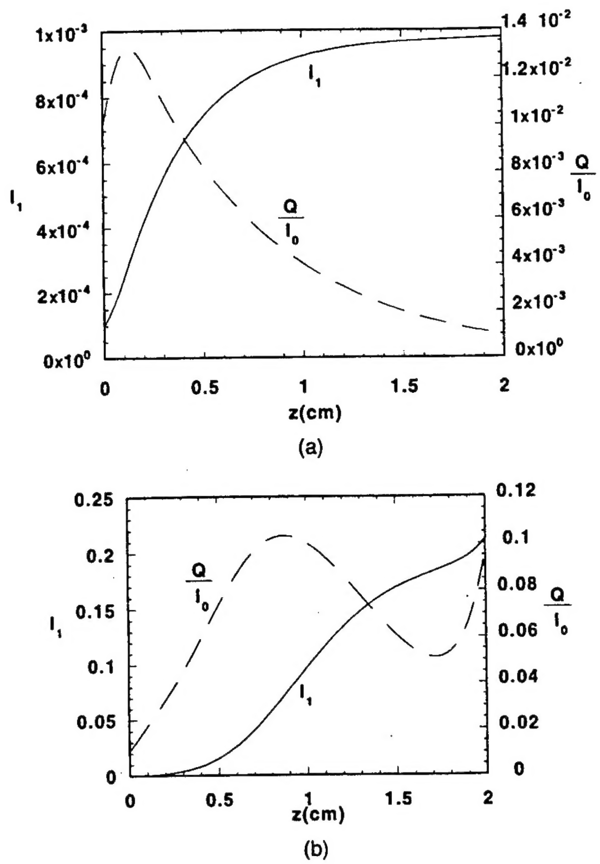


Fig. 7. Signal intensity $I_1(z)$ (solid curve) and the grating profile $\mathbf{Q}(z)/I_0(z)$ (dashed curve) as functions of the position z inside the photorefractive medium for an incident intensity ratio $\beta = 10^{-4}$ and coupling constants (a) $\gamma = 10\ \mathrm{cm}^{-1}$ and (b) $\gamma = 20\ \mathrm{cm}^{-1}$.

incident pump wave. Furthermore, the bandwidth of the amplified signal wave decreases as the length of the photorefractive grating increases. In Fig. 8(a), since the coupling constant is small, the pump-wave depletion is small ($\gamma = 10\ \mathrm{cm}^{-1}$) and the spectrum of the transmitted pump wave is almost the same as that of the incident pump wave. In Fig. 8(b), since the coupling constant is large ($\gamma = 20\ \mathrm{cm}^{-1}$), the pump depletion is significant. The central part of the incident pump-wave spectrum is depleted most significantly, and therefore the transmitted pump wave has a different spectrum from the incident pump wave. According to the spectra shown in Fig. 8, the case considered in Fig. 7(a) and Fig. 8(a) is in the nondepleted-pump regime, and the case considered in Fig. 7(b) and Fig. 8(b) is in the depleted-pump regime, although both cases seem to be in the nondepleted-pump regime when we look only at the signal intensity. In Figs. 9(a) and 9(b) we show with solid curves the normalized mutual coherence of the two waves as a function of position z, again using the same sets of parameters as those used to obtain Figs. 7(a) and 7(b), respectively. For comparison we show in these two figures with dashed curves the normalized mutual coherence of the two waves as a function of position z without coupling. In Fig. 9(a) the normalized mutual coherence of the two waves is the same with coupling as that without coupling in the region $z < 0$ and is increased in the region $z > L$ owing to coupling; in Fig. 9(b) the normalized mutual coherence of the two waves is decreased in the region close to plane $z = 0$. The increase of mutual coherence in both Figs. 9(a) and 9(b) can be attributed to the reflection of the strong pump wave in the direction of the weak signal wave by the stationary photorefractive index grating. The decrease of mutual coherence in Fig. 9(b) can be attributed to the spectral-filtering effect of the photorefractive index grating on the pump wave. There is no decrease of mutual coherence in the region close to plane $z = 0$ in Fig. 9(a) because pump depletion and therefore the spectral-filtering effect on the pump wave are negligible in this case.

Third, when a laser beam enters a photorefractive crystal, scattering occurs because of surface pits, imperfec-

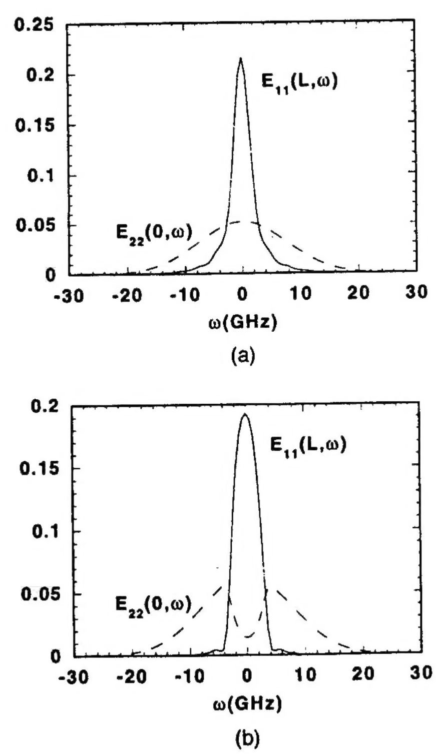


Fig. 8. Normalized spectra of the amplified signal wave (solid curve) and the depleted pump wave (dashed curve) at their respective exit planes for an incident intensity ratio $\beta = 10^{-4}$ and coupling constants (a) $\gamma = 10\ \mathrm{cm}^{-1}$ and (b) $\gamma = 20\ \mathrm{cm}^{-1}$.

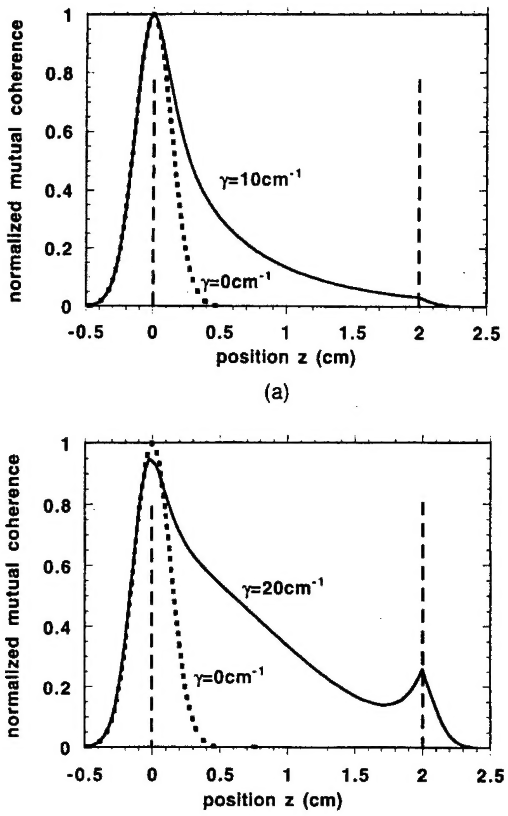


Fig. 9. Normalized mutual coherence of the two waves as functions of position z for an incident intensity ratio $\beta = 10^{-4}$ and for coupling constants (a) $\gamma = 10\ \text{cm}^{-1}$ (solid curve) and $\gamma = 0\ \text{cm}^{-1}$ (dashed curve) and (b) $\gamma = 20\ \text{cm}^{-1}$ (solid curve) and $\gamma = 0\ \text{cm}^{-1}$ (dashed curve).

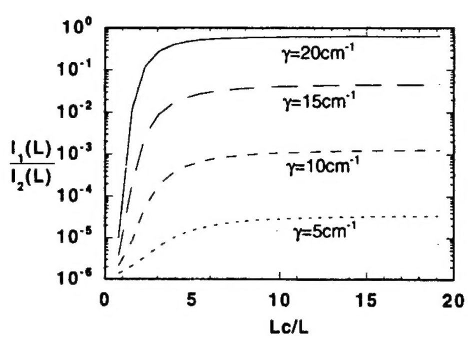


Fig. 10. Phase-conjugation reflectivity as a function of the coherence length of the incident beam for various value of the coupling constant γ.

tion, and defects in the crystal. The scattered light overlaps with the incident beam and they undergo two-wave mixing. Under the appropriate condition the scattered light can be amplified, leading to phenomena such as fanning and stimulated backscattering. In a manner very similar to stimulated Brillouin scattering the stimulated backward scattering in photorefractive media is a possible mechanism for self-pumped phase conjugation.[14,15] The general formulation developed in this paper can be employed to investigate the effect of coherence on self-pumped phase conjugation by **2k** gratings. We show in Fig. 10 the phase-conjugation reflectivity as a function of the coherence length of the incident beams for various value of the coupling constant γ. The parameters in this simulation are $n = 2.3$, $L = 0.72$ cm, $\alpha = 0.0\ \text{cm}^{-1}$, and $\beta = 1.0 \times 10^{-6}$. The time delay of the two waves is assumed to be zero at the signal-wave entrance plane ($z = 0$). The coherence length of the incident beams is related to the bandwidth as $L_c = 2\pi \times 0.664c/\delta\omega$. We note that the phase-conjugation reflectivity increases as the coherence length of the incident beam increases and reaches to a constant when the coherence length of the incident beam is much longer than the thickness of the photorefractive medium. We also note that the phase-conjugation reflectivity increases as the coupling constant increases.

4. EXPERIMENTS

The above theory is validated experimentally. The experimental setup is shown in Fig. 11. We utilized a 45°-cut $KNbO_3$:Co crystal (the c axis is in the horizontal plane leaning toward the $z = 0$ face of the crystal, and the b axis is in the vertical direction). The measured parameters of the crystal are $\gamma = 3.3\ \text{cm}^{-1}$, $\alpha = 0.5\ \text{cm}^{-1}$, $n = 2.3$, and $L = 0.72$ cm. A multimode argon laser operating at 514 nm is used as the laser source with a measured FWHM bandwidth of 1.83 GHz. The extraordinary polarization of the laser wave is used in the experiment. As illustrated in Fig. 11, the signal wave and the pump wave, obtained by splitting the argon laser wave, propagate contradirectionally into the $KNbO_3$:Co crystal. The incident intensity ratio of the two waves is $\beta = 0.00151$. The power of the pump wave is maintained at ~50 mW. The optical path difference of the two waves at the signal-wave incident plane $z = 0$ is denoted as $\Delta L = L_2 - L_1$. To monitor the mutual coherence between the signal wave and the pump wave at the output plane $z = L$, we employed another reference wave ($\mathbf{E}_{2\text{ref}}$) that was split from the pump wave $\mathbf{E}_2$. The optical path difference of $\mathbf{E}_1$ and $\mathbf{E}_{2\text{ref}}$ waves was adjusted to be the same as that of $\mathbf{E}_1$ and $\mathbf{E}_2$ waves at the output plane $z = L$. By a simple homodyne technique, the in-

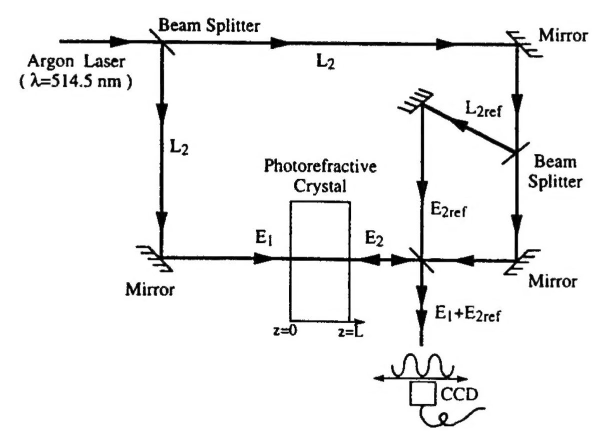


Fig. 11. Experimental setup. The distances L_1 and L_2 are the optical path length of the signal wave and the pump wave from laser source to the signal-wave incident plane $z = 0$, respectively. $L_{2\text{ref}}$ is the optical path length of reference wave $E_{2\text{ref}}$ from the laser source to the signal output plane $z = L$.

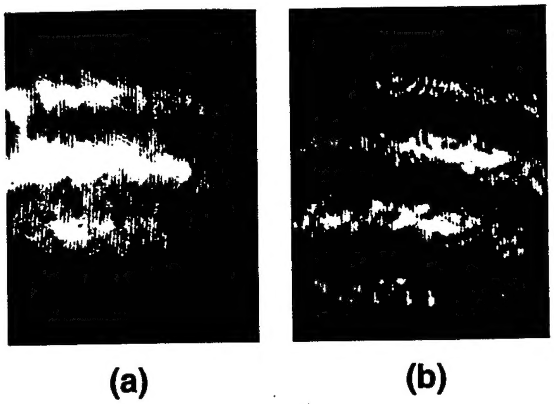


Fig. 12. Interference pattern of the signal wave and the reference wave at the output plane P1 (a) without photorefractive coupling and (b) with photorefractive coupling. Note the increase of fringe visibility that is due to the coupling.

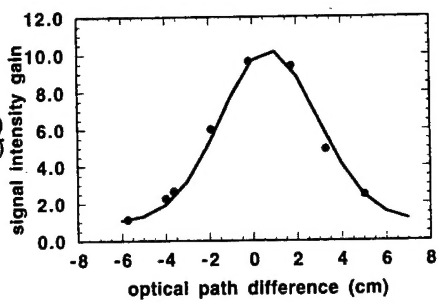


Fig. 13. Signal-intensity gain as a function of the optical path difference of the two waves at the signal-wave entrance plane ($z = 0$). The dots are experimental data, and the solid curve is the theoretical data.

terference fringes generated by $\mathbf{E}_1$ and $\mathbf{E}_{2\mathrm{ref}}$ waves were observed by a CCD camera at the output plane $P1$. The normalized mutual coherence $\gamma_{12}(L)$ can be estimated as $(I_{\max} - I_{\min})/[4(I_1 I_2)^{1/2}]$, where $(I_{\max} - I_{\min})$ is the amplitude of the fringes. In our experiment we monitored the interference pattern with and without pump beam $\mathbf{E}_2$. Figure 12 shows the interference patterns with a normalized mutual coherence $\Gamma_{12}(0, 0) \approx 0.43$ at $z = 0$ ($\Delta L = 4$ cm). The measured normalized mutual coherence increases from 0.19 to 0.7 at $z = L$. The intensity gain of the signal wave was also measured. Figure 13 shows the measurement of the intensity gain (dots) of the signal wave at the $z = L$ plane as a function of the optical path difference ΔL. Along with the data is the theoretical curve for the same parameters. An excellent agreement between theory and experiment was achieved.

5. CONCLUSIONS

In conclusion, we have investigated theoretically contradirectional two-wave mixing with partially coherent waves in photorefractive crystals. A general formulation based on the theory of statistical properties of linear systems is provided.[16] Previous results on several simplified cases are rederived as special cases of the general formulation so that we can get more insight into the general formulation as well as into the simplified cases. Results of numerical implementation of the general formulation are also provided for various coupling constants and various incident intensity ratios between the signal wave and the pump wave. We found that the mutual coherence between the signal wave and the pump wave can be both increased and decreased owing to coupling. We also found that both the strength and the length of the photorefractive-index grating increases as the coupling constant increases. A photorefractive index grating much longer than the coherence length of the incident waves can be formed when the coupling constant is large and the incident intensity ratio is small. Owing to the spectral-filtering effect of the photorefractive-index grating, the spectra of the two interacting waves will be altered as they pass through the photorefractive medium. The general formulation is further used to model the effect of partial coherence on self-pumped phase conjugation by a $2\mathbf{k}$ grating. The theoretical predictions are in excellent agreement with experimental measurements.

APPENDIX A: ERGODICITY PROPERTY

In this appendix we provide the derivation of Eq. (10). Equation (9) gives the dynamics of the photorefractive index grating. It can be rewritten into an integration form as

$$\mathbf{Q}(z, t) = \frac{1}{\tau_{\mathrm{ph}}} \int_{-\infty}^{t} \mathbf{E}_1(z, t')\mathbf{E}_2^*(z, t')\exp\left(\frac{t' - t}{\tau_{\mathrm{ph}}}\right)\mathrm{d}t'. \tag{A1}$$

Note that $1/\tau_{\mathrm{ph}}\int_{-\infty}^{t} \exp[(t' - t)/\tau_{\mathrm{ph}}]\mathrm{d}t' = 1$. According to Eq. (A1), the grating amplitude $\mathbf{Q}(z, t)$ is approximately the average value of $\mathbf{E}_1(z, t')\mathbf{E}_2^*(z, t')$ over a time period τ_{ph}.

Since the optical wave amplitudes $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ are stationary random processes, the ensemble average $\langle \mathbf{E}_1(z, t)\mathbf{E}_2^*(z, t)\rangle$ is independent of the time variable t. Taking the ensemble average of Eq. (A1), we can obtain

$$\langle \mathbf{Q}(z, t)\rangle = \langle \mathbf{E}_1(z, t)\mathbf{E}_2^*(z, t)\rangle \equiv \Gamma_{12}(z, 0). \tag{A2}$$

Equation (A2) shows that the ensemble average of the grating amplitude $\mathbf{Q}(z, t)$ is equal to the mutual coherence of the two waves.

In general, the grating amplitude $\mathbf{Q}(z, t)$ is also a random variable. It fluctuates around its ensemble average. The mean square value of this random fluctuation, also called the variance of the random variable $\mathbf{Q}(z, t)$, can be written as

$$\langle |\mathbf{Q}(z, t) - \langle \mathbf{Q}(z, t)\rangle|^2\rangle = \langle |\mathbf{Q}(z, t)|^2\rangle - |\langle \mathbf{Q}(z, t)\rangle|^2. \tag{A3}$$

By using Eq. (A1), we can write the first term on the right side of Eq. (A3) explicitly as

$$\langle |\mathbf{Q}(z, t)|^2\rangle = \frac{1}{\tau_{\mathrm{ph}}^2} \int_{-\infty}^{t}\int_{-\infty}^{t} \langle \mathbf{F}(z, t')\mathbf{F}^*(z, t'')\rangle \times \exp\left(\frac{t' - t}{\tau_{\mathrm{ph}}}\right)\exp\left(\frac{t'' - t}{\tau_{\mathrm{ph}}}\right)\mathrm{d}t'\mathrm{d}t'', \tag{A4}$$

where $\mathbf{F}(z, t) \equiv \mathbf{E}_1(z, t)\mathbf{E}_2^*(z, t)$ is a shorthand notation. Since the optical wave amplitudes $\mathbf{E}_1(z, t)$ and $\mathbf{E}_2(z, t)$ are stationary random processes, $\mathbf{F}(z, t)$ is also a stationary random process. Therefore the ensemble average $\langle \mathbf{F}(z, t')\mathbf{F}^*(z, t'')\rangle$ is a function of only two variables z and $t' - t''$. We define $\mathbf{R}(z, t' - t'') \equiv \langle \mathbf{F}(z, t')\mathbf{F}^*(z, t'')\rangle$ as a shorthand notation. With this relation we can change the integration arguments in Eq. (A4) from t' and t'' to $t_1 = t' + t'' - 2t$ and $t_2 = t' - t''$ and rewrite Eq. (A4) as

$$\langle |\mathbf{Q}(z, t)|^2\rangle = \frac{1}{2\tau_{\mathrm{ph}}^2}\int_{-\infty}^{0} \mathrm{d}t_1 \exp\left(\frac{t_1}{\tau_{\mathrm{ph}}}\right)\int_{t_1}^{-t_1} \mathrm{d}t_2 \mathbf{R}(z, t_2). \tag{A5}$$

Note that $1/(2\tau_{\mathrm{ph}}^2)\int_{-\infty}^{0}\mathrm{d}t_1 \exp(t_1/\tau_{\mathrm{ph}})\int_{t_1}^{-t_1}\mathrm{d}t_2 = 1.$ Substituting Eqs. (A2) and (A5) into Eq. (A3), we can rewrite the variance of the grating amplitude $\mathbf{Q}(z, t)$ as

$$\langle |\mathbf{Q}(z, t) - \langle \mathbf{Q}(z, t)\rangle|^2\rangle = \frac{1}{2\tau_{\mathrm{ph}}^2}\int_{-\infty}^{0} \mathrm{d}t_1 \exp\left(\frac{t_1}{\tau_{\mathrm{ph}}}\right)\int_{t_1}^{-t_1} \mathrm{d}t_2[\mathbf{R}(z, t_2) - |\Gamma_{12}(z, 0)|^2]. \tag{A6}$$

Remember that $\mathbf{F}(z, t)$ is a stationary random process. Let Δt be the minimum time delay that is necessary for $\mathbf{F}(z, t)$ and $\mathbf{F}(z, t \pm \Delta t)$ to be uncorrelated. For $|t_2| \geqslant \Delta t$, $\mathbf{R}(z, t_2) - |\Gamma_{12}(z, 0)|^2$ is equal to zero. For $|t_2| \leqslant \Delta t$, $\mathbf{R}(z, t_2) - |\Gamma_{12}(z, 0)|^2$ is a function of t_2. Since both $\mathbf{R}(z, t_2)$ and $|\Gamma_{12}(z, 0)|^2$ are of the order of $I_1(z)I_2(z)$, the upper bound of $\mathbf{R}(z, t_2) - |\Gamma_{12}(z, 0)|^2$ can be written as $mI_1(z)I_2(z)$, where m is a constant factor of the order of unity. With these estimations we can obtain from Eq. (A6)

$$\langle |\mathbf{Q}(z, t) - \langle \mathbf{Q}(z, t)\rangle|^2\rangle < mI_1(z)I_2(z)\frac{\Delta t}{\tau_{\mathrm{ph}}}. \tag{A7}$$

According to relation (A7), the fluctuation of the grating amplitude $\mathbf{Q}(z, t)$ decreases as the ratio $\Delta t/\tau_{\mathrm{ph}}$ decreases. When $\Delta t/\tau_{\mathrm{ph}} \ll 1$, we can neglect the fluctuation of the grating amplitude $\mathbf{Q}(z, t)$ and obtain Eq. (10), i.e.,

$$\mathbf{Q}(z, t) \approx \langle \mathbf{Q}(z, t)\rangle = \Gamma_{12}(z, 0). \tag{A8}$$

Usually Δt is of the order of the coherence time $(\delta\omega)^{-1}$ of the incident waves. In this case the condition for Eq. (A8) can also be written as $\delta\omega\tau_{\mathrm{ph}} \gg 1$.

In this appendix we have shown that the time average of the random variable $\mathbf{E}_1(z, t)\mathbf{E}_2^*(z, t)$ is equal to the ensemble average of the random variable $\mathbf{E}_1(z, t)\mathbf{E}_2^*(z, t)$. This type of property is referred to as ergodicity in statistics.

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REFERENCES

1. See, for example, P. Yeh, *Introduction to Photorefractive Nonlinear Optics* (Wiley, New York, 1993).
2. See, for example, P. Gunter and J.-P. Huignard, eds., *Photorefractive Materials and Devices I and II*, Vols. 61 and 62 of Topics in Applied Physics (Springer-Verlag, Berlin, 1988, 1989).
3. B. Fischer, S. Sternklar, and S. Weiss, "Photorefractive oscillators," IEEE J. Quantum Electron. **25**, 550 (1989).
4. V. Wang, "Nonlinear optical phase conjugation for laser systems," Opt. Eng. **17**, 267 (1978).
5. S. C. De La Cruz, S. MacCormack, J. Feinberg, Q. B. He, H. K. Liu, and P. Yeh, "Effect of beam coherence on mutually pump phase conjugators," J. Opt. Soc. Am. B **12**, 1363 (1995).
6. Q. B. He and P. Yeh, "Photorefractive mutually pumped phase conjugation with partially coherent beams," Appl. Phys. B **60**, 47 (1995).
7. R. Hofmeister, A. Yariv, and S. Yagi, "Spectral response of fixed photorefractive grating interference filters," J. Opt. Soc. Am. A **11**, 1342 (1994).
8. N. V. Bogodaev, L. I. Ivleva, A. S. Korshunov, N. M. Polozkov, and V. V. Shkunov, "Increase of light-beam coherence by two-wave mixing in photorefractive crystals," J. Opt. Soc. Am. B **10**, 2287 (1993).
9. X. Yi, S. H. Lin, P. Yeh, and K. Y. Hsu, "Contradirectional two-wave mixing with partially coherent waves in photorefractive crystals," Opt. Lett. **21**, 1123 (1996).
10. See, for example, J. W. Goodman, *Statistical Optics* (Wiley, New York, 1985), Chap. 3.
11. P. Yeh, "Fundamental limit of the speed of photorefractive effect and its impact on device applications and material research," Appl. Opt. **26**, 602 (1987).
12. R. Saxena, F. Vachss, I. McMichael, and P. Yeh, "Diffraction properties of multiple-beam photorefractive gratings," J. Opt. Soc. Am. B **7**, 1210 (1990).
13. R. Saxena, C. Gu, and P. Yeh, "Properties of photorefractive gratings with complex coupling constants," J. Opt. Soc. Am. B **8**, 1047 (1991).
14. T. Y. Chang and R. W. Hellwarth, "Optical phase conjugation by backscattering in barium titanate," Opt. Lett. **10**, 108 (1985).
15. B. Ya. Zel'dovich, V. I. Popovichev, V. V. Ragul'skii, and F. S. Faizullov, "Connection between the wavefronts of the reflected and exciting light in SBS," Zh. Éksp. Teor. Fiz. **15**, 160 (1972).
16. A. W. Snyder, D. J. Mitchell, and Y. S. Kivshar, "Unification of linear and nonlinear wave optics," Mod. Phys. Lett. B **9**, 1479 (1995).